Simulation for Composite Cylindrical Pressure Hull using Progressive Response Surface Method

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Abstract. Response surface method (RSM) as a fitting method had been applied to composite cylindrical buckling problem. There are little researches focusing on the accuracy of the RSM. For most cylinder buckling problems, only angles of the fiber orientation had been considered as the important variables. This paper proposes a progressive response surface method (RSM) to approximate the buckling critical pressure for composite cylindrical pressure hull taking ply angles and ply thicknesses into account. This method is adopted to replace the finite element method (FEM) to rise the calculation efficiency, and the fitting model result in this method can be proved to be effective and accurate compared with 6 different RSM models. The results can demonstrate its accuracy and feasibility under ply angles and thicknesses for composite cylindrical buckling problem.

Keywords: Progressive response surface method; Buckling problem; Composite pressure hull; ply angles; ply thicknesses.

1. Introduction

Due to the high strength to weight ratio and resistance to corrosion, the laminated underwater composite submersible has been extensively applied to ocean exploration. Generally, the submersible is typically cylindrical shape and the laminated shell is one of the normal types of composite structures. Due to prolonged exposure to external water pressure, buckling of composite cylindrical shell may occur [1]. In recent decades, there are lots of researchers conducted study concerning buckling performance of the cylindrical shell in order to attain a better buckling solution of cylinder [2–6].

To investigate the buckling of the cylinder, the characteristics of laminated composite shell play very significant role and a quantity of methods have been proposed to address this issue. Hsuan-Teh Hu and Su Su Wang [7] performed a sequential linear programming method to solve the buckling optimization of composite shell with and without cutouts. Chul-Jin Moon [6] treated winding angles as one of the most buckling and failure characteristics for thick-walled composite cylindrical shell under external hydrostatic pressure. Elsayed Fathallah and Hui Qi [8,9] performed the optimal design of laminated composite underwater vehicle considering buckling strength, failure strength and Tsai-Wu Criteria using ANSYS software. Stefano Francesco Pitton carried out the buckling optimization of thin-walled with general path of fibers' formulation which is composed by fibers angles using artificial intelligence techniques and finite element method [10].

Most of the above studies employed the finite element method (FEM) which may cost unimaginable time under intelligence optimization algorithm. To overcome these drawbacks, the response surface approximation as quite efficient method was proposed for buckling performance to reduce the computationally expensive cost in stacking sequence optimization process by Akira Todoroki and Tetsuya Ishikawa [11]. In addition, M. Abouhamze and M. Shakeri established the artificial neural networks (ANN) used in the buckling conditions to advance the speed of the optimization procedure [12]. Similarly, to predict the buckling load of composite panels, the artificial neural networks were constructed for testing and training data based on FEM by Upendra K. Mallela and Akhil Upadhyay [13]. Diab W. Abueidda conducted the convolutional neural network approximate model to predict the two-dimensional phase composite mechanical properties and integrated this model with the genetic algorithm optimization [14].

Unfortunately, in order to meet the requirements of approximation accuracy, more training data has to be selected. Although these scholars can obtain relatively precise approximate models through sufficient training samples, it will consume more time and computing resources. An improved regression of the response surface method had been designed to save much more time in optimal stacking sequence and ply angles of composite laminates by Ya-Jung Lee and Ching-Chieh Lin, and this technique had proved to be a quick and accurate method [15].

However, in article [15], only angles of the fiber orientation had been taken into consideration. In the practical application, both the fiber orientation angles and layer thicknesses are very sensitive to the cylinder buckling performance [9]. Therefore, a more progressive response surface method is produced to integrate the angles of orientation and the layer thicknesses for more efficient fitting and optimization.

The rest of this paper can be arranged as follows. In Section 2, a progressive response surface method will be introduced in order to improve the effective of the traditional response surface method. It will be presented the method of the buckling simulation of pressure composite cylindrical shell in Section 3. After both the traditional response surface method models and the progressive response surface method models are established, a comparison of these models will be executed to determine the accuracy and effectiveness of the progressive RSM model in Section 4. Finally, the conclusion will be made in Section 5.

2. Progressive response surface method

2.1 Traditional response surface method (RSM)

The RSM was produced to construct polynomial approximate model for the objective by Box and Hunter [1]. To construct the RSM, the Design of experiment (DOE) is needed to develop a good relationship between input data and output response. Besides the Latin hypercube design (LHD) which can provide much design space information is used to attain the original sample data. Suppose the input sample variables as $x_1, x_2, \cdots, x_n$ and response data as y. Then the RSM model can expressed as:

$$\boldsymbol{y}(\boldsymbol{x}) = \tilde{\boldsymbol{y}}(\boldsymbol{x}) + \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{y}(\boldsymbol{x})$ indicates the unknown objective function, $\tilde{\boldsymbol{y}}(\boldsymbol{x})$ is the polynomial function of $\boldsymbol{x}$, and ε_i illustrates numerical convergence error or experimental measurement error which is assumed to be normal distribution with mean of zero and variance of σ^2. Usually, the polynomial approximate function may adopt linear, second-order, or higher order polynomial models. If the polynomial function selects linear model, this model will be illustrated below:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \tag{2}$$

where,

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}, \quad \boldsymbol{X} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1n} \\ 1 & x_{21} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{k1} & \cdots & x_{kn} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix}$$

The least squares method which is employed in the linear regression model to assess the least squares coefficients, $\boldsymbol{b}$.

$$L=\sum_{i=1}^{n}\varepsilon_i^2=\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}=\boldsymbol{y}^T\boldsymbol{y}-2\boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{y}+\boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{\beta} \tag{3}$$

Then the least squares method must content Equation (4).

$$\left.\frac{\partial L}{\partial \beta}\right|_b=-2\boldsymbol{X}^T\boldsymbol{y}+2\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{b}=0 \tag{4}$$

This formula can be simplified as:

$$\boldsymbol{b}=\left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}\boldsymbol{X}^T\boldsymbol{y} \tag{5}$$

Then the fitted linear model can be presented:

$$\tilde{\boldsymbol{y}}=\boldsymbol{X}\boldsymbol{b} \tag{6}$$

Generally, the response surface method function is unknown, and it must be fitted as Equation (6). In the practical engineering, the high order polynomial may be adopted such as second order, third order and forth order model (see Equation 7, 8 and 9).

$$\tilde{y}=\beta_0+\sum_{i=1}^{n}\beta_i x_i+\sum_{i=1}^{n}\beta_{ii}x_i^2+\sum_{i=1}^{n}\sum_{j>i}^{n}\beta_{ij}x_i x_j \tag{7}$$

$$\tilde{y}=\beta_0+\sum_{i=1}^{n}\beta_i x_i+\sum_{i=1}^{n}\beta_{ii}x_i^2+\sum_{i=1}^{n}\sum_{j>i}^{n}\beta_{ij}x_i x_j+\sum_{i=1}^{n}\beta_{iii}x_i^3 \tag{8}$$

$$\tilde{y}=\beta_0+\sum_{i=1}^{n}\beta_i x_i+\sum_{i=1}^{n}\beta_{ii}x_i^2+\sum_{i=1}^{n}\sum_{j>i}^{n}\beta_{ij}x_i x_j+\sum_{i=1}^{n}\beta_{iii}x_i^3+\sum_{i=1}^{n}\beta_{iiii}x_i^4 \tag{9}$$

2.2 Progressive response surface method

The regression analysis was proposed in the response surface to approximate the laminated composite structures considering ply angles as design variables by Ya-Jung Lee and Ching-Chieh Lin [2,3]. In their research, the trigonometric functions were employed as base regression, and the response of the regression was perfectly fitted. This regression analysis stimulates a progressive response surface method (progressive RSM) which converts the angle parameters to the form of trigonometric function under the composite problem involving the angle of orientation. The transformation from traditional RSM to progressive RSM can be seen in the Figure 1.

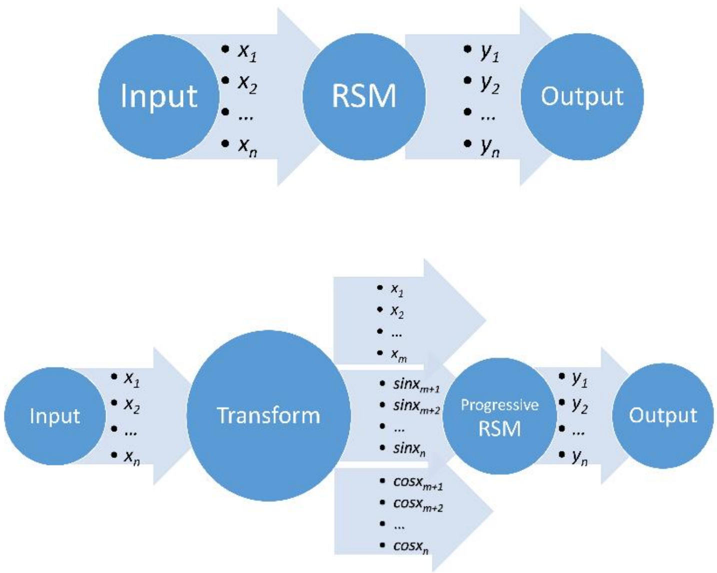

Fig. 1. The diagram of progressive RSM

On the other hand, only the angle of ply orientation was taken into account as the input parameter, and another important factor called thickness of ply was neglected in the above research. Consequently, in this paper, the ply angles and the ply thickness will be considered to construct the progressive response surface model simultaneously to improve the accuracy of the traditional response surface method. In this method, the second, third and fourth order progressive RSM formula can be presented:

$$\begin{aligned}\tilde{y} = B_0 &+ \sum_{i=1}^{m} B_i x_i + \sum_{i=m+1}^{n} B_i \sin x_i + \sum_{i=m+1}^{n} B_{i+m} \cos x_i + \\ &\sum_{i=1}^{m}\sum_{j=1}^{m} C_{ij} x_i x_j + \sum_{i=m+1}^{n}\sum_{j=m+1}^{n} D_{ij} \sin x_i \sin x_j + \\ &\sum_{i=m+1}^{n}\sum_{j=m+1}^{n} E_{ij} \sin x_i \cos x_j + \sum_{i=m+1}^{n}\sum_{j=m+1}^{n} F_{ij} \cos x_i \cos x_j\end{aligned} \tag{10}$$

$$\begin{aligned}\tilde{y} = B_0 &+ \sum_{i=1}^{m} B_i x_i + \sum_{i=m+1}^{n} B_i \sin x_i + \sum_{i=m+1}^{n} B_{i+m} \cos x_i + \\ &\sum_{i=1}^{m}\sum_{j=1}^{m} C_{ij} x_i x_j + \sum_{i=m+1}^{n}\sum_{j=m+1}^{n} D_{ij} \sin x_i \sin x_j + \\ &\sum_{i=m+1}^{n}\sum_{j=m+1}^{n} E_{ij} \sin x_i \cos x_j + \sum_{i=m+1}^{n}\sum_{j=m+1}^{n} F_{ij} \cos x_i \cos x_j + \\ &\sum_{i=1}^{m} G_i x_i^3 + \sum_{i=m+1}^{n} G_i \sin^3 x_i + \sum_{i=m+1}^{n} G_{i+m} \cos^3 x_i\end{aligned} \tag{11}$$

$$\begin{aligned}\tilde{y} = B_0 &+ \sum_{i=1}^{m} B_i x_i + \sum_{i=m+1}^{n} B_i \sin x_i + \sum_{i=m+1}^{n} B_{i+m} \cos x_i + \\ &\sum_{i=1}^{m}\sum_{j=1}^{m} C_{ij} x_i x_j + \sum_{i=m+1}^{n}\sum_{j=m+1}^{n} D_{ij} \sin x_i \sin x_j + \\ &\sum_{i=m+1}^{n}\sum_{j=m+1}^{n} E_{ij} \sin x_i \cos x_j + \sum_{i=m+1}^{n}\sum_{j=m+1}^{n} F_{ij} \cos x_i \cos x_j + \\ &\sum_{i=1}^{m} G_i x_i^3 + \sum_{i=m+1}^{n} G_i \sin^3 x_i + \sum_{i=m+1}^{n} G_{i+m} \cos^3 x_i + \\ &\sum_{i=1}^{m} H_i x_i^4 + \sum_{i=m+1}^{n} H_i \sin^4 x_i + \sum_{i=m+1}^{n} H_{i+m} \cos^4 x_i\end{aligned} \tag{12}$$

For the purpose of comparison with each kind of response surface model, there are six types of models will be produced (Equation 7-12). The Model 1 is a second order traditional response surface model with 231 terms for 20 independent variables (Equation 7). And the Model 2 and 3 are third and forth order traditional response surface model with 251 and 271 terms for the same 20 variables (Equation 8 and 9). The Model 4, 5 and 6 are second, third and fourth order progressive response surface model using sine and cosine functions with 496, 526 and 556 terms for the same variables (Equation 10, 11 and 12).

3. Buckling simulation of composite cylinder

Consider an orthotropic composite cylindrical pressure shell whose length is 800 mm and radius is 150 mm in the experiment of Ouellette, P. and Chul-Jin Moon [4,5]. In the research of Elsayed Fathallah [6], there are 4 layers with about 5 mm total thickness can withstand the pressure under 4000 m depth, and the general thickness is 0.1-1 mm for each layer [7]. For most composite cylindrical pressure shell problem, the number of conventional layers is 10 [8,9]. That means the total thickness of 2-10 mm can be applied to the practical submersible engineering. And according to the filament winding process, the angle and thickness of each layer can be illustrated by θ_i and t_i respectively as shown in Figure 2. Moreover, the value range of the ply angle and ply thickness can be expressed as:

$$\begin{cases} -90° \le \theta_i \le 90° \\ 0.2\,mm \le t_i \le 1\,mm \\ i = 1, 2, \cdots, n \end{cases} \tag{13}$$

Then the response of this model is buckling critical pressure P_{cr}. The material of composite cylinder will adopt carbon fiber and epoxy resin, and the material properties can be shown in Table 1.

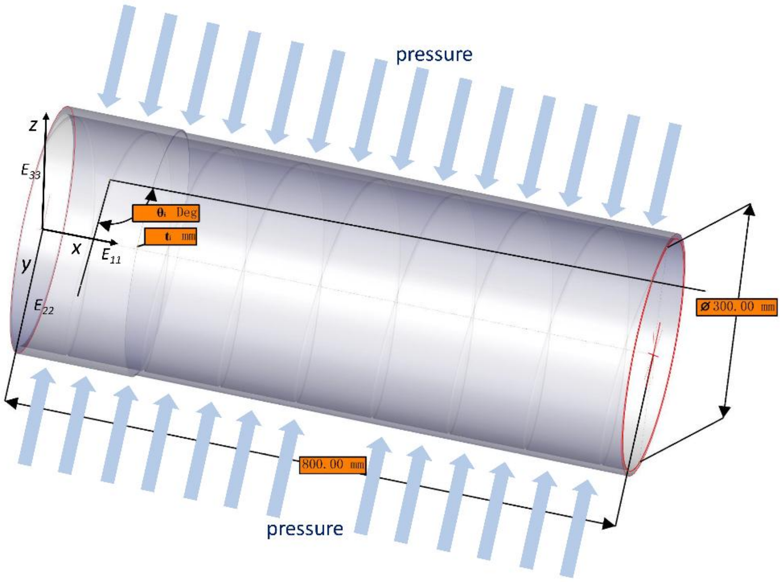

Fig. 2. The diagram of cylindrical shell model

Table 1.Material properties of composite cylinder

Property	Symbol and value	Unit
Elastic modulus	E_{11}=121 , E_{22}=8.6 , E_{33}=8.6	Gpa
Poisson's ratio	v_{12}=0.253, v_{13}=0.253, v_{23}=0.421	-
Shear modulus	G_{12}=3.35 , G_{13}=3.35 , G_{23}=2.68	Gpa

4. Results and discussions

As mentioned in Section 2, the traditional and progressive RSM can be established by combining LHD and FEM using Ansys and Matlab commercial software. To test the accuracy of each models, the multiple correlation coefficient R^2 and mean relative error MRE can be proposed as follows.

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \left(y_i - \tilde{y}_i \right)^2}{\sum_{i=1}^{n} \left(y_i - \mu(y) \right)^2} \tag{21}$$

$$MRE = \frac{1}{n} \sum_{i=1}^{n} \frac{\left| y_i - \tilde{y}_i \right|}{\left| y_i \right|} \tag{22}$$

10 layer thicknesses and 10 layer angles were selected as input parameters of the model, and the output response is buckling critical pressure. The Figure 3 and Figure 4 are error responses comparing four models. The Figure 3 expresses the error distribution of initial points, and Figure 4 expresses the error distribution of test points. From these two figures, Model 1, 2 and 3 have almost the same error distributions in pressure composite cylinder problem, even though they have different orders, and Model 4, 5 and 6 have more accuracy than traditional RSM models.

In addition, from Table 2, the multiple correlation coefficient R^2 of initial points and the mean relative error of test points MRE are also nearly the same which are about 0.86 and 12% respectively. That means Model 1, 2 and 3 approximate poorly for both initial points and test points. On the other hand, Model 4, 5 and 6 have the more precise fitting result which can reduce the error by about 5.1% compared to the traditional RSM models.

Moreover, Model 5 and 6 have less error than Model 4 for initial points, but the gap between Model 5 (or 6) and Model 4 is very small (about 0.24 %). And Model 4 has the best multiple correlation coefficient which means the best fitting performance. That is to say Model 4 has sufficient precision and fitting degree.

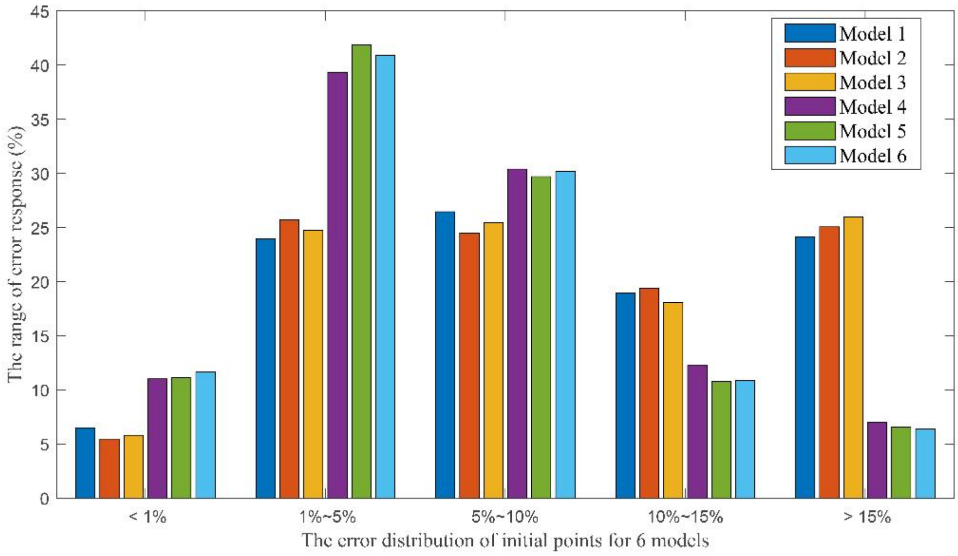

Fig. 3. The error distribution of initial points

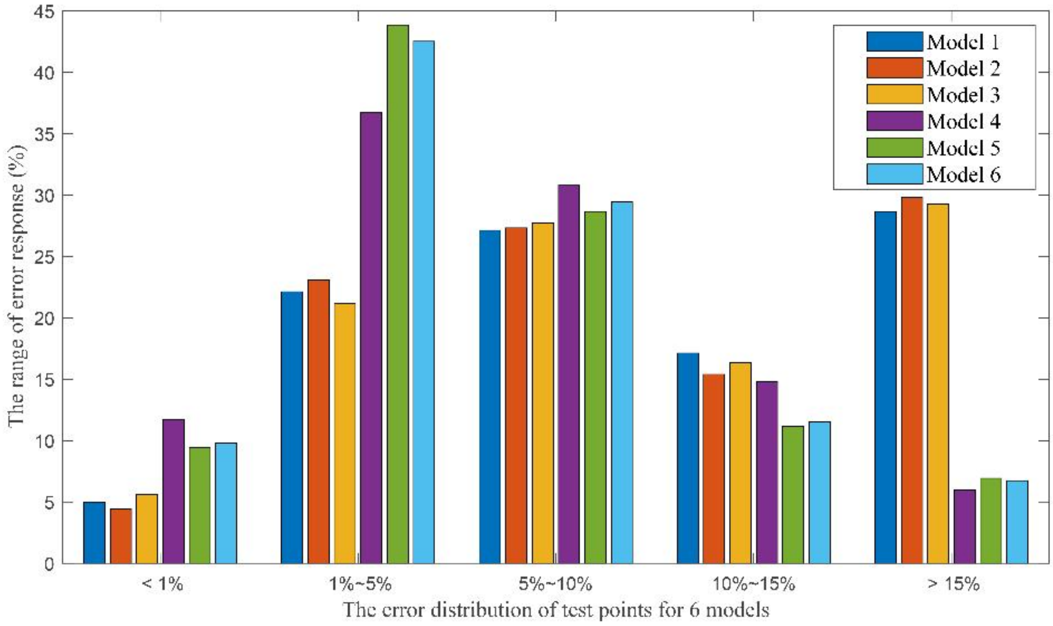

Fig. 4. The error distribution of test points for four models

Table 2.The statistics of 4 models for fitting performance of initial points

Statistics	Model 1	Model 2	Model 3	Model 4	Model 5	Model6
R^2	0.8659	0.8553	0.8569	0.9791	0.961	0.9611
MRE	0.1079	0.1145	0.1144	0.0635	0.0611	0.061

The Figure 5 describes the error distribution results for different numbers of test points. Normally, the fewer the sample points are, the better the fitting accuracy is. However, the fewer the test points are, the poorer the fitting accuracy is [3]. Generally, the fitting performance rises as the number of initial points increases which can be shown in Table 3. It can be seen that when the number of initial points exceed 2000, the error distribution of test points stops decreasing. That is to say 2000 can be the effective number of initial sample points under 10 ply angles and 10 ply thicknesses parameters.

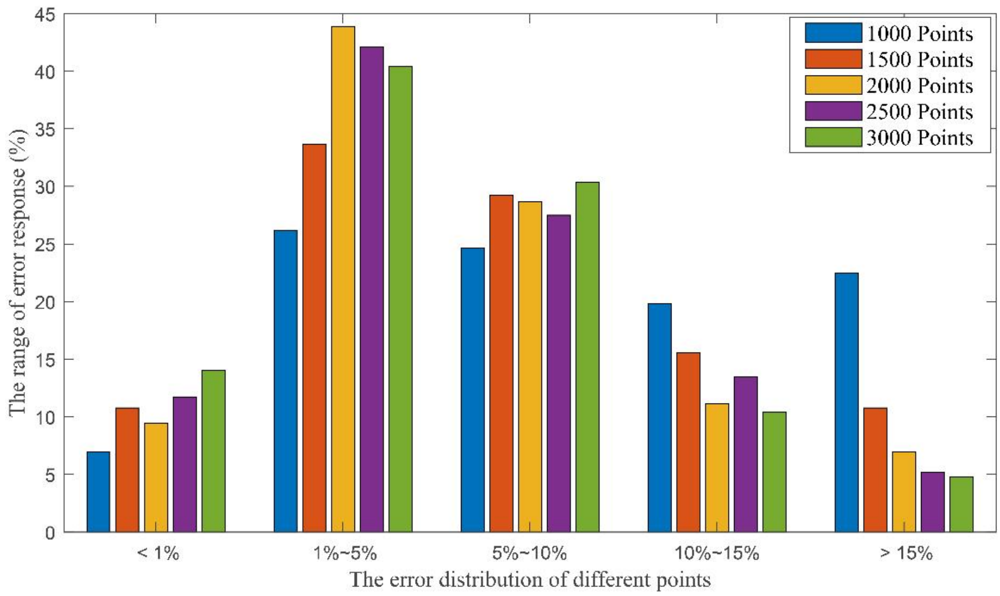

Fig. 5. The error distribution of different points

Table 3.The statistics of different numbers for fitting performance of test points

Statistics	1000 points	1500 points	2000 points	2500 points	3000 points
R^2	0.885	0.9413	0.9623	0.9647	0.9656
MRE	0.113	0.0796	0.0635	0.0617	0.0607

5. Conclusions

In this paper, both layer thickness and angle are taken into consideration in composite cylindrical pressure shell to construct a progressive RSM model. And this progressive RSM model had been compared with other traditional models of different orders, and it is proved to be quite good method. However, this method can only be applied to the problems containing the angles of orientation. It is an obvious limitation in this progressive RSM. Then a robust based optimization was carried out to ensure the robustness of optimization solution based on the progressive RSM model. The following conclusions are drawn.

1. A progressive RSM is proposed to fit the buckling critical pressure P_{cr} related to both the ply thickness and ply angles of the composite cylindrical pressure shell using the trigonometric functions. And this method has a more accurate fitting result which can save about 5.1 % error compared with other traditional RSM models.

2. Even though Model 1, 2 and 3 have different orders, there is no evidence that higher order functions are more accurate for the buckling critical pressure problem. And the same thing happens in Model 4, 5 and 6. Consequently, it is better to construct the progressive RSM model with second order model.

3. For 20 input variables, it can reach a relatively precise fitting model by 2000 or more sample points. Even with more sample points, the approximate model does not get more obvious accuracy.

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