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# Based on MLM Slamming Model: Higher-Order Solutions Derivation

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**Abstract.** The field of hydrodynamics has witnessed growing interest, both domestically and internationally, in optimizing slamming theory models. This shift has moved from the traditional Wagner's slamming theory towards more advanced theoretical frameworks that account for nonlinear effects. In this context, this study focuses on deriving higher-order solutions based on the Modified Logvinovich Model (MLM) slamming theory, a prominent representative within these improved paradigms. The original MLM model only incorporated the leading-order term, which, although a useful approximation, is insufficient for capturing the full range of nonlinear effects. Thus, the first-order solution derived from this leading term serves merely as an approximation and should not be considered the definitive solution governing the MLM model. The exploration of higher-order solutions and the expansion of the model's dimensionality to better account for nonlinear effects is an essential task in current hydrodynamic research. This study investigates the application of MLM slamming theory to wedge-water entry, with a particular focus on higher-order solutions. By introducing higher-order mathematical series expansions and extending the dimensional framework of the theory, we aim to enhance the accuracy and applicability of slamming predictions for objects entering water. The second-order MLM slamming solutions derived here provide a more comprehensive understanding of the slamming phenomenon. Additionally, the use of complex series superposition in the higher-order solution further improves the model's capability to describe the behavior of wedge entry at large incline angles, offering a more precise characterization of the slamming process.

**Keywords:** MLM slamming theory; higher-order solutions; object entry into water.

## 1. Introduction

Water slamming is a common fluid structure coupling physical phenomenon in nature, such as ship surface slamming in adverse sea conditions, and seaplane landing. This high-speed and brief relative motion will generate a huge impact load, namely a slamming load. Banging loads can cause serious damage to marine structures and even result in catastrophic irreversible accidents. Therefore, the systematic theoretical research on the problem of water slamming has important practical engineering significance.

Given the complex flow mechanics issues involved in the slamming process, most theoretical model studies have been simplified to some extent. Among them, the earliest was the two-dimensional wedge-shaped impact model proposed by von Karman [1], which simplified the process of aircraft colliding with water. This theoretical model equates a wedge body to a flat plate and can calculate the impact force on the wedge body, but it is powerless to handle the surface pressure distribution and does not consider the real factor of liquid level rise. Wagner [2] further improved the slamming theory to address the issue of liquid level rise, which can provide pressure distribution and more accurate slamming force. However, for such a strongly nonlinear physical problem, the pressure solving process in this theory only considers the linear part and ignores all nonlinear terms.

Based on the Wagner model, there are many improved theoretical models to enhance the solution of the slamming problem [3]. The nonlinear term plays a crucial role in analyzing pressure distribution and slamming force, and the original Logvinovich model (OLM) compensates for this

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regret on the basis of the Wagner model, which can better predict wedges with smaller inclined lift angles [11]. In order to broaden the application range of oblique lift angle, the improved Logvinovich model (MLM) improves the calculation of slamming force by retaining second-order terms related to oblique lift angle in the pressure distribution [12].

However, it is worth noting that the original MLM method only considered the first order in asymptotic analysis, but the first-order MLM slamming solution derived from the leading-order term is merely an approximation within the realm of nonlinear effects and should not serve as the sole solution governing the MLM slamming model. In this study, we extensively extended the MLM slamming model and provided a second-order explicit analytical solution, which was validated through CFD numerical calculations. In addition, we have provided some reflections on future research on higher dimensional improvements.

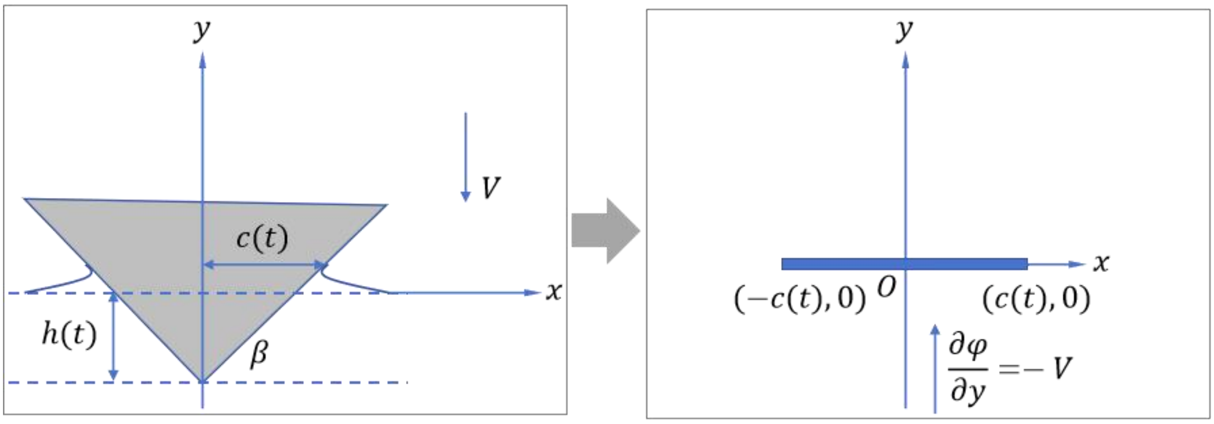
# 2. High Order MLM Model Derivation

## 2.1 Original MLM Model

### 2.1.1 Sub heading

The original MLM slamming model was based on the Wagner slamming model, which was derived by considering the nonlinear term in the Bernoulli equation and applying asymptotic analysis to first order expansion of velocity potential.

Continuing the research object of wedges under the Wagner model, considering the effect of rising free liquid level, a flat plate is used instead of an object entering water. Therefore, the wetting half width of the artificial wedges is c(t), as shown in the Fig. 1.




(a) Vertical entry of wedges into water (b) Equivalent flat plate hypothesis model

Fig. 1 Sketch of Wagner slamming model for wedge-shaped body

The Wagner slamming model only considers the term $-\rho\varphi_t$ in the Bernoulli equation, while the original MLM slamming model considers nonlinear terms (ignoring gravity and surface tension terms) on this basis, i.e. the pressure expression is

$$p(x,y,t) = -\rho\left(\varphi_t + \frac{1}{2}|\nabla\varphi|^2\right) \tag{1}$$

where $\rho$ is water density, $\varphi(x,y,t)$ is the velocity potential to be solved in the MLM slamming model. The original MLM slamming model introduces a small parameter $\varepsilon = f(L)/L$ and selects the characteristic parameters in Table 1 to obtain a dimensionless velocity potential control equation as follows.

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$$\begin{cases} \Delta\bar{\varphi}=0 & \left(\text{in } \bar{\Omega}\left(\bar{t}\right)\right) \\ \dfrac{\partial\bar{\varphi}}{\partial\bar{t}}+\varepsilon\dfrac{1}{2}\left|\nabla\bar{\varphi}\right|^2=0 & \\ \dfrac{\partial\bar{\eta}}{\partial\bar{t}}+\varepsilon\dfrac{\partial\bar{\eta}}{\partial\bar{x}}\dfrac{\partial\bar{\varphi}}{\partial\bar{x}}=\dfrac{\partial\bar{\varphi}}{\partial\bar{y}} & \left(\bar{y}=\varepsilon\bar{\eta}\left(\bar{x},\bar{t}\right)\right) \\ \dfrac{\partial\bar{\varphi}}{\partial\bar{y}}=\varepsilon\bar{f}'\left(\bar{x}\right)\dfrac{\partial\bar{\varphi}}{\partial\bar{x}}-\dfrac{\partial\bar{\eta}}{\partial\bar{t}} & \left(\bar{y}=\varepsilon\left[\bar{f}\left(\bar{x}\right)-\bar{h}\left(\bar{t}\right)\right]\right) \\ \bar{\varphi}\to 0 & \left(\bar{x}^2+\bar{y}^2\to\infty\right) \end{cases} \tag{2}$$

where $\bar{h}=h/\varepsilon L,\bar{f}\left(\bar{x}\right)=f\left(x/L\right)/\varepsilon L,\bar{\varphi}(\bar{x},\bar{y},\bar{t})=\varphi(x,y,t)/V_0L.$

$\bar{y}=y/\varepsilon L=\bar{\eta}\left[x/L,V_0t/\varepsilon L\right]$ is the expression describing the shape of the free liquid surface.

Based on the above dimensionless process, the asymptotic analysis and Taylor expansion of the velocity potential can be performed to obtain the first-order velocity potential of the MLM slamming model[12]:

$$\phi_{MLM}^{1st}\left(x,t\right)=-\dot{h}\left(t\right)\sqrt{c^2\left(t\right)-x^2}-\dot{h}\left(t\right)\left[f(x)-h(t)\right] \tag{3}$$

The specific velocity potential is related to water entry depth $h(t)$, velocity $\dot{h}(t)$, water entry half-width $c(t)$ and shape function $f(x)$. Substituting the velocity potential (3) into the pressure distribution expression (1), the first-order pressure distribution expression of MLM slamming model can be obtained:

$$P_{MLM}^{1st}\left(x,t\right)=\rho\dot{h}^2\left(t\right)\left[\frac{\dot{c}}{\dot{h}}\frac{c}{\sqrt{c^2-x^2}}-\frac{1}{2}\frac{c^2}{c^2-x^2}\frac{1}{1+f_x^2}-\frac{1}{2}\frac{f_x^2}{1+f_x^2}+\frac{\ddot{h}}{\dot{h}}\left(\sqrt{c^2-x^2}+f-h\right)\right], \tag{4}$$

where $h(t)$ corresponds to $h(0)=0$ at the moment $t=0$.

The first-order slamming force expression of MLM slammning model is given by integrating the pressure distribution (4):

$$F_{MLM}^{1st}(t)=2\int_0^{a(t)}P_{MLM}^{1st}(x,t)dx \tag{5}$$

### 2.2 Second-Order MLM Model

The original MLM slamming model only gives the expression of the main order solution, i.e. the first-order solution, which improves the application range of the deadrise angle compared with the Wagner slamming model to a certain extent. However, the core idea of MLM slamming model is to make use of the progressive analysis idea introduced by parameters and the Talyor expansion principle in the process of solving the velocity potential.

From the perspective of mathematical thinking, the first-order solution should not be the only normal solution of the MLM slamming model. The derivation of the high order theoretical solution will provide a new way to expand the understanding and application of the MLM slamming model. In this study, the original MLM slamming model is extended. In the derivation of the asymptotic analysis method, the higher order is considered. The core idea of which is to expand the velocity potential to the second-order form:

$$\phi(x,t)=V_0L\left[\bar{\varphi}_1\left(\bar{x},\varepsilon\left[\bar{f}\left(\bar{x}\right)-\bar{h}\left(\bar{t}\right)\right],\bar{t}\right)+\varepsilon\bar{\varphi}_2\left(\bar{x},\varepsilon\left[\bar{f}\left(\bar{x}\right)-\bar{h}\left(\bar{t}\right)\right],\bar{t}\right)+O\left(\varepsilon^2\right)\right], \tag{6}$$

where, the Taylor expansion of $\bar{\varphi}_1$ and $\bar{\varphi}_2$ is performed at $y=0$, respectively. The expression is as follows:

$$\bar{\varphi}_1\left(\bar{x},\varepsilon\left[\bar{f}\left(\bar{x}\right)-\bar{h}\left(\bar{t}\right)\right],\bar{t}\right)=\bar{\varphi}_1\left(\bar{x},0,\bar{t}\right)+\varepsilon\frac{\partial\bar{\varphi}_1\left(\bar{x},0,\bar{t}\right)}{\partial\bar{y}}\left[\bar{f}\left(\bar{x}\right)-\bar{h}\left(\bar{t}\right)\right]+O\left(\varepsilon^2\right), \tag{7}$$

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$$\overline{\varphi}_2\left(\overline{x},\varepsilon\left[\overline{f}\left(\overline{x}\right)-\overline{h}\left(\overline{t}\right)\right],\overline{t}\right)=\overline{\varphi}_2\left(\overline{x},0,\overline{t}\right)+\varepsilon\frac{\partial\overline{\varphi}_2\left(\overline{x},0,\overline{t}\right)}{\partial\overline{y}}\left[\overline{f}\left(\overline{x}\right)-\overline{h}\left(\overline{t}\right)\right]+O\left(\varepsilon^2\right), \tag{8}$$

Further, the second-order velocity potential expression of MLM slamming model can be obtained by combining the boundary conditions.

$$\phi_{MLM}^{2nd}\left(x,t\right)=-\dot{h}\left(t\right)\sqrt{c^2\left(t\right)-x^2}+\left[f(x)-h(t)\right]\left\{f'(x)\left[\dot{h}x\sqrt{c^2-x^2}\right]-\dot{h}\left(t\right)\right\}. \tag{9}$$

The second-order pressure distribution expression of MLM slamming model:

$$P_{MLM}^{2nd}\left(x,t\right)=P_{MLM}^{1st}-\rho\begin{bmatrix}\dfrac{\partial\phi_{MLM}^{Exc}}{\partial t}+\dfrac{f_x\dot{h}}{1+f_x^2}\dfrac{\partial\phi_{MLM}^{Exc}}{\partial x}\\+\dfrac{1}{1+f_x^2}\dfrac{\partial\phi_{MLM}^{1st}}{\partial t}\dfrac{\partial\phi_{MLM}^{Exc}}{\partial t}+\dfrac{1}{2\left(1+f_x^2\right)}\left(\dfrac{\partial\phi_{MLM}^{Exc}}{\partial t}\right)^2\end{bmatrix}, \tag{10}$$

Where $\phi_{MLM}^{1st}$ is given by Eq. (3)， $\phi_{MLM}^{Exc}$ is calculated by

$$\phi_{MLM}^{Exc}=f'(x)\left[f(x)-h(t)\right]\left[\dot{h}x\sqrt{c^2-x^2}\right]. \tag{11}$$

The second-order slamming force model can be obtained by integrating Eq.(11)

$$F_{MLM}^{2nd}(t)=2\int_0^{a(t)}P_{MLM}^{2nd}(x,t)dx. \tag{12}$$

## 3. Higher-Order Solutions Verification

In order to quantitatively evaluate the sensitivity of the deadrise angle to the high order solution, the vertical water entry velocity of the wedge is selected as $V=0.61m/s$ and the water density is $\rho=10^3kg/\mathrm{m}^3$. Table 1 shows the specific proportions of the first-order pressure $\phi_{MLM}^{1st}$ and the excess of second-order pressure comparing to the first-order solution $P_{MLM}^{Exc}$ under five different deadrise angle cases. It is clearly seen from Table 1 that $P_{MLM}^{Exc}$ increases rapidly with the increase of $\beta$, and the corresponding five groups of working conditions are 0.950%, 4.255%, 10.448%, 21.053% and 62.500%. The influence of the proportion of the second-order solution beyond the first-order solution is getting larger and larger, which cannot be ignored.

Table 1. Proportion sensitivity of $P_{MLM}^{1st}$ and $P_{MLM}^{Exc}$ to the large deadrise angle $\beta$.

| $\beta(°)$ | $P_{MLM}^{1st}$ Proportion | $P_{MLM}^{Exc}$ Proportion |
|---|---|---|
| 20 | 99.050% | 0.950% |
| 30 | 95.745% | 4.255% |
| 40 | 89.522% | 10.448% |
| 50 | 78.947% | 21.053% |
| 60 | 37.500% | 62.500% |

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The comparison of high and low order MLM solutions with CFD numerical results for $\beta = 56^{\circ}$ is investigated. As is shown in Fig. 2. It can be observed that the CFD numerical solution at the 0-0.1m position fluctuates near the high order MM pressure distribution solution. However, the lower order MLM pressure distribution solution is much lower than the numerical solution. Although the high order MLM slamming theory solution has errors of 1.5% for $\beta = 56^{\circ}$ compared to the CFD numerical results, the accuracy of the high order theoretical solution is sufficiently high compared to the 40% error of the lower order MLM solution. In addition, it is emphasized that CFD results did not consider the small lifting part of the free surface when converting stable pressure values in numerical simulations, which may be the reason for the slight difference between pressure values and the high order MLM solutions.

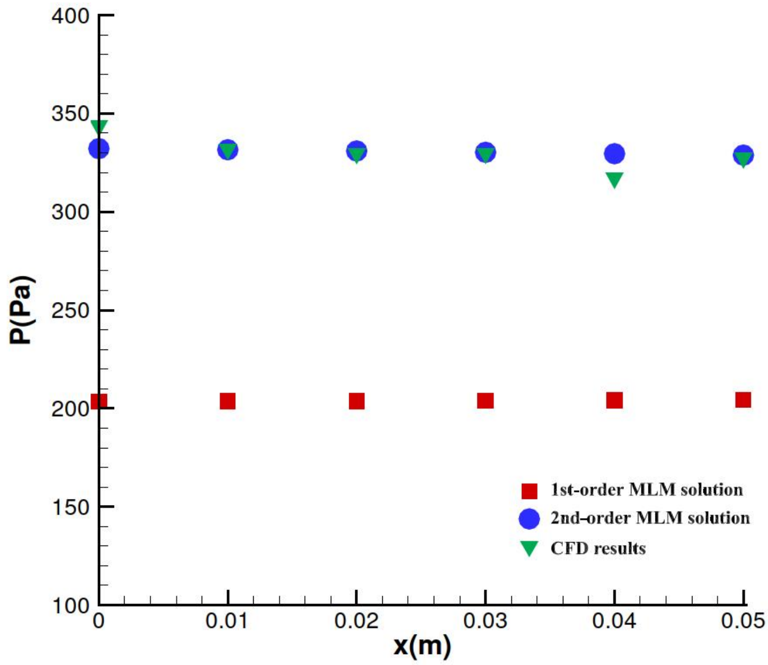


Fig. 2 Comparison of high and low order solutions with CFD numerical results for $\beta = 56^{\circ}$

In fact, the high order MLM slamming model derived from two dimensions in this study can theoretically be extended to three dimensions. At least for the three-dimensional model of the top wave, the high order slamming force can be evaluated by fast algorithm at different positions. This will be generalized in future work.

## 4. Conclusions

The principle of the original MLM slamming model is investigated, and the explicit solution of the high order MLM slamming model, including the velocity potential and pressure distribution, is innovatively derived. The high order slamming force can be quickly obtained by integrating the pressure distribution.

Sensitivity analysis was conducted on the big deadrise angles of the high order and lower order explicit pressure solutions. It was found that the proportion of $P_{MLM}^{Exc}$ begins to increase, and the larger the $\beta$, the greater the proportion, reaching up to 62%. This means that the part of the high order solution cannot be ignored. Besides, the obtained high order MLM slamming model expression is compared with the lower order solution and CFD numerical results. It was found that CFD numerical results were more consistent with the high order MLM solution. The accuracy of the lower order MLM solutions cannot be guaranteed for large $\beta$, with an error of up to 40%. The derivation of the high order MLM solutions expands the applicability of the MLM slammning model to larger deadrise angles, providing a new technique for the rapid assessment of slammning

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problems in structures with large deadrise angle. It is also emphasized that in the future, the high order MLM solution model is expected to be extended to higher-dimensional slamming computation.

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