

Research on Ship Wake–Sea Surface Interaction Modeling Based on Gaussian Energy Coupling

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Abstract: The optical detection accuracy of ship wakes strongly depends on the precision of wake–sea surface coupling modeling. Most existing studies ignore the energy interaction between the two and adopt a direct superposition approach, which results in significant deviation from the actual marine environment. This paper focuses on the research framework of “wake distribution characteristics–sea surface modeling–coupling superposition–detection impact.” First, the three-dimensional distribution characteristics of ship wakes in length, width, and depth are systematically reviewed, and the quantitative relationships between key parameters such as total wake length, effective detection length, diffusion angle, and ship type (length, draft), as well as sailing speed, are clarified. Second, a three-dimensional sea surface geometric model is constructed based on the Elfouhaily spectrum and the Longuet-Higgins directional distribution function, revealing the modulation patterns of wind speed and fetch on sea surface roughness. Third, to overcome the limitations of existing superposition methods, a Gaussian-energy-based wake–sea coupling modeling method is proposed, in which each wake point is regarded as an energy source, and a Gaussian function is used to describe its influence weight on the surrounding sea surface. Finally, multi-case simulations verify that this method can accurately reproduce the modulation effect of the sea surface on wake wave height—the larger the ship speed and hull size, the more significant the wake fluctuations; meanwhile, increasing sea surface wind speed enhances background interference to the wake signal. The results provide theoretical support and a modeling method for improving the anti-interference capability and target recognition accuracy of ship wake optical detection.

Keywords: Wake distribution characteristics; Elfouhaily spectrum; Longuet-Higgins directional distribution function; sea surface geometric modeling

1. Introduction

A ship wake is a characteristic fluid trace generated during navigation by propeller cavitation and hull wave breaking. It typically lasts 10–15 minutes and serves as a core reference for tracing and identifying ship targets using optical, acoustic, and other techniques, with important value in both military and civilian fields. Militarily, it is a key method for anti-submarine warning and ship stealth evaluation; in civilian applications, it supports shipping safety monitoring, illegal vessel tracking, and marine environment disturbance analysis.

Optical detection, with its advantages of high spatial resolution and fast real-time response, has become a mainstream technique for wake detection. However, in actual ocean environments, wake signals are always modulated and disturbed by sea surface waves (gravity waves and capillary waves). Neglecting their coupling leads to large deviations in wake shape reconstruction, significantly reducing recognition rates and anti-interference ability of detection systems. Therefore,

constructing a physically consistent “wake–sea surface” coupling geometric model and clarifying its modulation effect on optical detection have both theoretical and engineering significance.

The three-dimensional distribution characteristics of wakes form the modeling foundation. Existing studies have preliminarily established the relationship between key parameters and ship attributes: Ma et al.^[0] experimentally verified that the total wake length is 20–50 times the ship length, with initial depth close to the draft; Chen et al.^[2] established a linear relationship between effective detection length and sailing speed, providing reference for optical detection range design. For typical Kelvin wakes, researchers have confirmed their sinusoidal periodic structure and the positive correlation between period and speed, but most studies focus on calm water environments and lack parameter corrections under sea state disturbances.

Sea surface modeling is mainly based on random rough surface theory, with the wave spectrum function as the key descriptor of wave energy distribution. The unified sea spectrum model proposed by Elfouhaily et al.^[3] realizes multi-sea-state simulation by decomposing long- and short-wave curvature spectra, making it the current mainstream method. Earlier PM and JONSWAP spectra^[5] apply to fully developed and limited-fetch seas respectively, but fail to simultaneously represent energy distributions of long (gravity) and short (capillary) waves. Regarding directional distribution functions, the Longuet-Higgins function^[6] has been widely used for constructing three-dimensional sea surfaces because it accurately reflects wind-direction modulation of wave energy; however, most existing works focus on single wind-driving parameters and lack coupled analysis of wind speed and fetch.

Currently, wake–sea surface modeling mostly adopts the “direct wave height superposition” method, in which the total wave height at the same spatial location equals the algebraic sum of wake and sea surface wave heights. Although simple, this method ignores the interaction between wakes as energy disturbance sources and sea waves—in reality, wake fluctuations modify surrounding sea surface energy distribution via fluid dynamic effects. Direct superposition fails to reproduce the sea surface’s smoothing or amplifying modulation of wakes, leading to deviations exceeding 30% compared to satellite remote sensing observations.

In summary, three major shortcomings exist in current research: (1) Superposition methods do not consider energy interactions between wake and sea surface, only using algebraic addition, inconsistent with real fluid disturbance mechanisms. (2) Correlations between wake parameters (diffusion angle, effective detection length) and sea states (wind speed, fetch) remain unclear, and sea surface modeling parameterization lacks systematic review. (3) No quantitative relationship has been established between coupled model parameters and optical scattering signals, limiting direct support for detection system optimization.

To address these issues, this study will focus on four aspects: (1) Systematically analyze the length, width, and depth distribution characteristics of ship wakes. (2) Construct a three-dimensional sea surface model based on the Elfouhaily–Longuet-Higgins (ELH) framework and quantify the influence of wind speed and fetch on sea surface fluctuations. (3) Propose a Gaussian-energy-based superposition method, describing wake point-source influence weights on the sea surface using Gaussian functions, achieving physically consistent coupling modeling. (4) Through multi-condition simulations, analyze the impacts of ship type, sailing speed, and sea state parameters on the superposition model, and reveal its modulation effect on optical detection.

2. Distribution Characteristics of Ship Wakes

Ship wakes, formed by propeller cavitation and hull wave breaking during navigation, are dynamic targets whose distribution characteristics depend on ship parameters, sailing state, and water environment. Their core features can be summarized from three dimensions—length, width, depth—along with sectional differences and influencing factors.

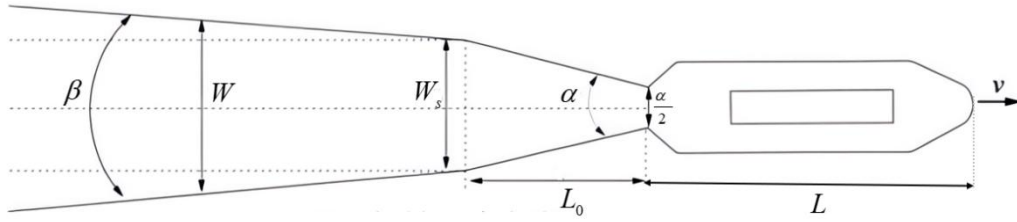


Fig. 1 Ship wake width

Fig. 1 shows the plan view of a ship wake. W_s is the wake width. L , B , and v denote ship length, beam, and speed. α is the diffusion angle of the wake.

The distribution characteristics of a wake usually refer to its thickness, effective detection length, and width⁰.

2.1 Wake Length Characteristics

Wake length refers to the effective trailing distance behind the hull, including total length and effective detection length:

The total wake length is generally 20–50 times the ship length, with strong persistence in water. It can last 10-15 minutes, providing a time window for detection.

The effective detection length is the segment detectable by laser or other instruments, directly correlated with ship speed^[2], expressed as:

$$L_A = C_A \cdot v \quad (1)$$

where L_A is the detectable wake length, C_A is a constant representing detection capability, and v for ship speed. Higher speed usually yields longer effective detection length.

2.2 Wake Width Characteristics

Based on cross-sectional measurements, the initial wake width is about half the ship beam, then expands outward at a constant diffusion angle α , as shown in Fig. 1. The diffusion angle is independent of ship speed.

When the wake length extends to L_0 (determined by ship speed), the diffusion angle changes to another angle β . Thus, wake width can be expressed as:

$$W(L) = \begin{cases} 2L \tan\left(\frac{\alpha}{2}\right) + \frac{B}{2} & L \leq L_0 \\ W_0 + 2(L - L_0) \tan\left(\frac{1}{2}\beta\right) & L > L_0 \end{cases} \quad (2)$$

where $\tan(\alpha/2) = (W_0 - B/2)/2L_0$, W_0 is wake width at L_0 . The initial diffusion angle α ranges between 40°-60°, and β does not exceed 1°. For simplicity, the 4D bubble distribution model adopts $\beta = 1^\circ$, reducing the expression to:

$$W(L) = \begin{cases} \frac{L}{L_0} \left(W_0 - \frac{B}{2} \right) + \frac{B}{2} & L \leq L_0 \\ W_0 + 0.017(L - L_0) & L > L_0 \end{cases} \quad (3)$$

2.3 Wake Depth Characteristics

The initial wake depth is approximately the ship draft DDD , as shown in Fig. 2.

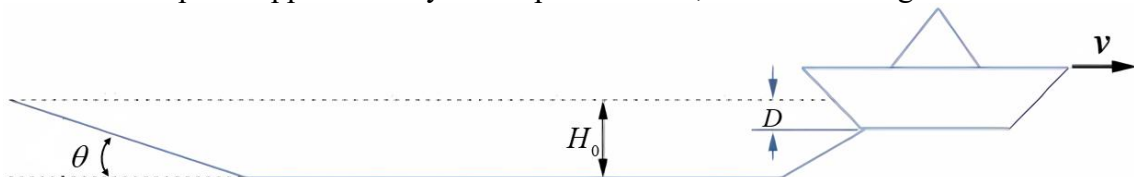
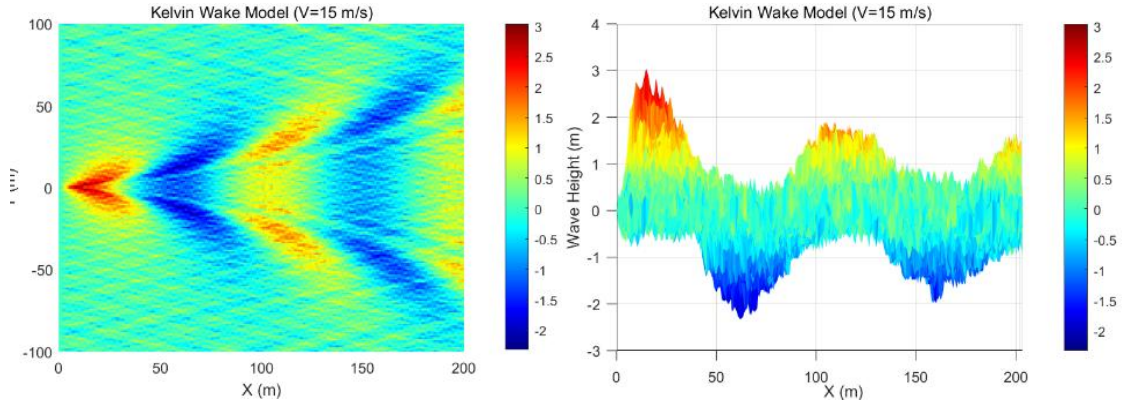


Fig. 2 Ship wake depth

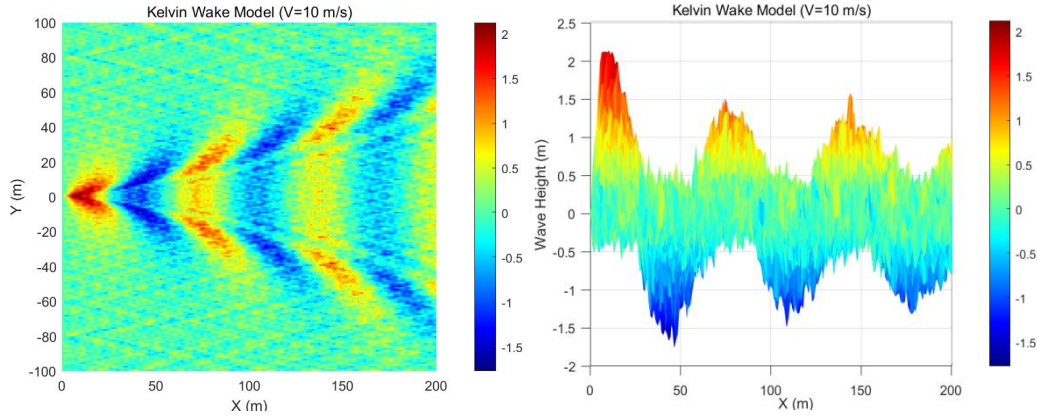
After the ship moves about one ship length L_{ship} , the wake depth increases to H_0 , approximately 1.5-2.0 times the draft. It remains nearly constant until the ship has advanced about 10 lengths, after which the depth decreases gradually at a small floating angle ($\sim 0.1^\circ$). Letting $H_0 = kD$, where k depends on ship type, the wake depth in the bubble distribution model with $\theta = 0.1^\circ$ simplifies to:

$$H(L) = \begin{cases} \frac{(k-1)DL}{L_{\text{ship}}} + D & L < L_{\text{ship}} \\ kD & L_{\text{ship}} \leq L \leq 10L_{\text{ship}} \\ kD - 0.0018(L - 10L_{\text{ship}}) & L > 10L_{\text{ship}} \end{cases} \quad (4)$$

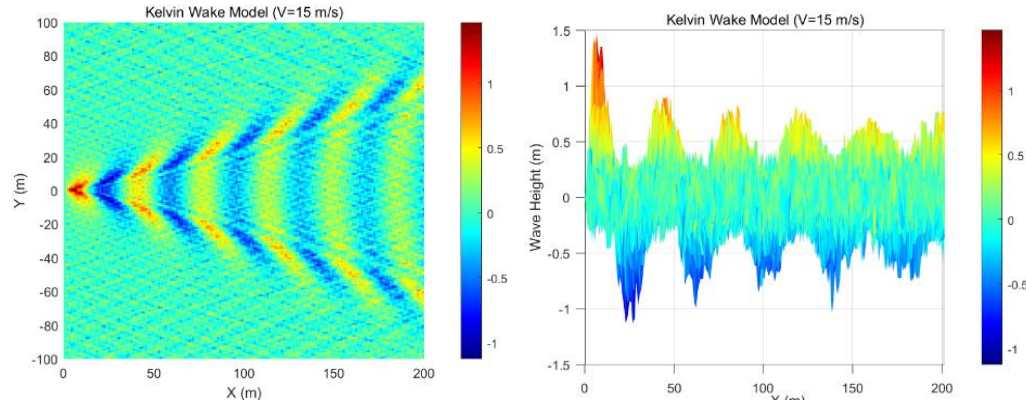
2.4 Simulation Results



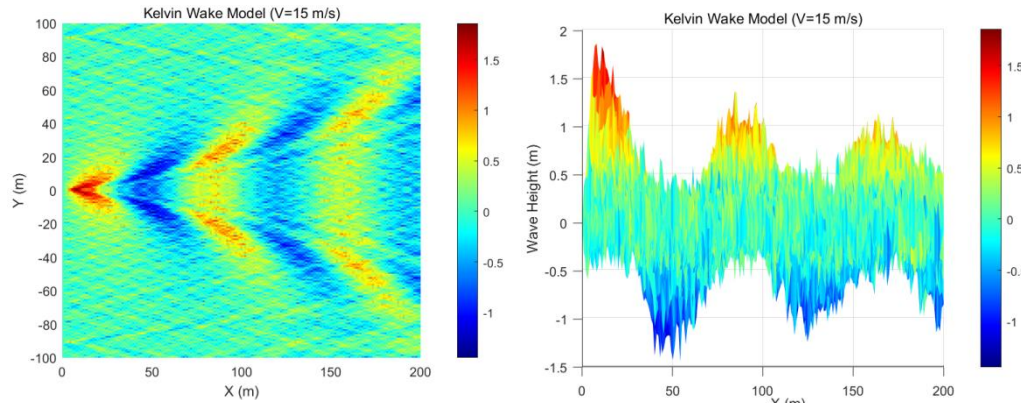
(a) $L=153\text{m}$, $V=15\text{m/s}$



(b) $L=153\text{m}$, $V=10\text{m/s}$



(c) $L=70\text{m}$, $V=10\text{m/s}$



(d) $L=70\text{m}$, $V=15\text{m/s}$

Fig. 3 Kelvin wakes under different ship types and speeds

The ship speeds in Fig. 3(a) and (b) are $V_1=15\text{m/s}$ and $V_2=10\text{m/s}$, respectively. From the cross-sectional views of the wakes, it is evident that, under the same conditions, ship speed has a significant influence on the wake pattern. The higher the speed, the stronger the wake fluctuations. When the speed is 15m/s , the fluctuation amplitude near the stern is about 2.5m ; when the speed is 10m/s , the amplitude is about 1.5m . The Kelvin wake clearly exhibits a sinusoidal periodic structure, and the higher the speed, the longer the period. The ship parameters for Fig. 3(a) and (b) are as follows: (a) $L=153\text{m}$, $B=20.4\text{m}$, $D=6.3\text{m}$; (b) $L=70\text{m}$, $B=10\text{m}$, $D=3.5\text{m}$.

From the cross-sectional views of the wakes, it is also evident that, at the same speed, ship size has a certain effect on the wake pattern. The larger the ship volume, the more intense the wake fluctuations. In Fig. 3(b), the near-stern wake amplitude is about 1.5m , while in Fig. 3(c) it is about 1m . Moreover, the wake period of ship (b) is shorter than that of ship (c).

In fact, for ships moving in the ocean, when the ship speed is relatively low and the sea state is relatively high, the wave height variation of the Kelvin wake is significantly affected by the ocean background. Therefore, the influence of the sea surface on wakes must be taken into account.

3. Sea Surface Modeling and Superposition of Ship Wake–Sea Surface Geometric Models

3.1 Sea Surface Modeling

Sea surface modeling is based on random rough surface theory. The core idea is to use a wave spectrum function combined with a directional distribution function, and to construct a three-dimensional sea surface geometric model through linear superposition and other methods, with emphasis on the modulation effect of wind forcing on sea surface morphology.

(1) Wave Spectrum Model: Elfouhaily Spectrum

The wave spectrum is the Fourier transform of the correlation function of wave height fluctuations, describing the distribution of wave energy with respect to wavenumber. In this study, the Elfouhaily spectrum^{[3][4]} is adopted (a unified model that incorporates and modifies the PM spectrum, JONSWAP spectrum^[5], and laboratory-measured data), which is suitable for simulating multiple sea states. Its expression and core components are as follows:

The Elfouhaily spectrum divides wave energy into two parts: long waves (dominated by gravity waves) and short waves (dominated by capillary waves). Its general expression is:

$$\Psi_E(k) = \frac{1}{k^3} [B_L(k) + B_S(k)] \quad (5)$$

Where $B_L(k)$ and $B_S(k)$ are the long-wave curvature spectrum and short-wave curvature spectrum, respectively:

$$B_L(k) = \frac{1}{2} \alpha_p \frac{c(k_p)}{c(k)} F_p \quad (6)$$

$$\alpha_p = 0.006 \Omega^{1/2} \quad (7)$$

The inverse wave age is defined as:

$$\Omega = u_{10} / c(k_p) \quad (8)$$

where c_p is the phase velocity:

$$c(k) = \sqrt{g / k(1 + k^2 / k_m^2)} \quad (9)$$

$$F_p = L_{PM} J_p \exp \left[-\frac{\Omega}{\sqrt{10}} \left(\sqrt{\frac{k}{k_p}} - 1 \right) \right] \quad (10)$$

In the Elfouhaily spectrum, the first two terms of F_p can be derived from the L_{PM} and J_p forms in the JONSWAP spectrum. Depending on the range of peak inverse wave age, J_p is redefined. The relevant parameters are:

$$J_p = \gamma^B \quad (11)$$

$$B = \exp \left\{ -\frac{\left[\left(k / k_p \right)^{1/2} - 1 \right]^2}{2\sigma_J^2} \right\} \quad (12)$$

$$\gamma = \begin{cases} 1.7, & \Omega_c \leq 1 \\ 1.7 + 6 \log(\Omega_c), & 1 < \Omega_c < 5 \\ 2.7 \Omega_c^{0.57}, & \Omega_c \geq 5 \end{cases} \quad (13)$$

$$\sigma_J = \begin{cases} 0.08(1 + 4\Omega_c^{-3}), & \Omega_c < 5 \\ 0.16, & \Omega_c \geq 5 \end{cases} \quad (14)$$

$$\Omega_c = 0.84 \left\{ \tanh \left[(\tilde{X} / X_0)^{0.4} \right] \right\}^{0.75}, \quad X_0 = 2.2 \times 10^4 \quad (15)$$

The short-wave curvature spectrum $B_s(k)$ is given by:

$$B_s(k) = \frac{1}{2} \alpha_m \frac{c(k_m)}{c(k)} F_m \quad (16)$$

$$F_m = \exp \left[-\frac{1}{4} \left(1 - \frac{k}{k_m} \right)^2 \right] \quad (17)$$

where α_m is the generalized equilibrium parameter for small-scale waves:

$$\alpha_m = \begin{cases} 1 + \ln \left[\frac{u_f}{c(K_m)} \right], & u_f \leq c(K_m) \\ 1 + 3 \ln \left[\frac{u_f}{c(K_m)} \right], & u_f > c(K_m) \end{cases} \quad (18)$$

$$c(K_m) = (2g / k_m) \approx 0.23m / s \quad (19)$$

u_f is the friction velocity at the sea surface.

(2) Directional Distribution Function: Longuet-Higgins Function

The directional distribution function, also called the angular distribution function, reflects the modulation of wave energy by wind direction. Among the commonly used functions for sea surface simulation is the Longuet-Higgins distribution function^[6], with its single-sided form often applied:

$$\Phi_{LH}(k, \varphi) = G(s) \left[\cos \left(\frac{\varphi - \varphi_w}{2} \right) \right]^{2z} \quad (20)$$

The correlation coefficient is:

$$s = 1 - \frac{1}{\ln 2} \ln \left[\frac{1 - \Delta(k)}{1 + \Delta(k)} \right] \quad (21)$$

$$G(s) = 1 / \int_{-\pi}^{\pi} \cos^{2z} \left(\frac{\varphi}{2} \right) d\varphi \quad (22)$$

The upwind-crosswind ratio $\Delta(k)$ depends on friction velocity and phase speed:

$$\Delta(k) = \tanh \left\{ \frac{\ln 2}{4} + 4 \left[\frac{c(k)}{c(k_p)} \right]^{-2.5} + 0.13 \left[\frac{u_f}{c(k_m)} \right] \left[\frac{c(k_m)}{c(k)} \right]^{2.5} \right\} \quad (23)$$

where $c(k_m)=23\text{cm/s}$. This applies to wind speeds at 10 m height $u_{10}<17.2\text{m/s}$. The friction velocity u_f is given by:

$$u_f = C_{10}^{-1.2} u_{10}, \quad C_{10} = (0.8 + 0.064 u_{10}) \times 10^{-3} \quad (24)$$

(3) Implementation of Sea Surface Geometric Modeling

Using the Elfouhaily spectrum Ψ_E and the single-sided Longuet-Higgins function Φ_{LH} , the ELH sea spectrum model is obtained:

$$S_{ELH}(k, \varphi) = \frac{1}{k} \Psi_E(k) \Phi_{LH}(k, \varphi - \varphi_w) \quad (25)$$

where Ψ_E 、 Φ_{LH} correspond to Equations (6) and (21).

The sea surface is constructed as the superposition of multiple random cosine components, each with random amplitude, wavenumber, and phase, generally following a Rayleigh distribution. Experimental data show that cosine components with very large or very small wavenumbers contribute little to the energy, which concentrates within a limited range.

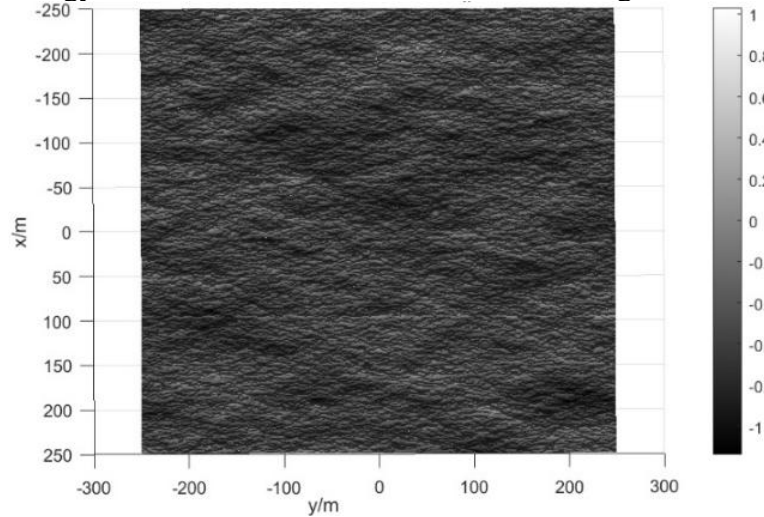
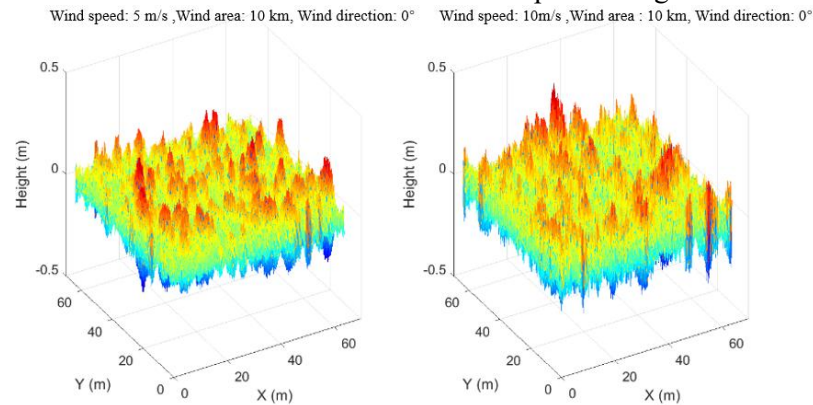
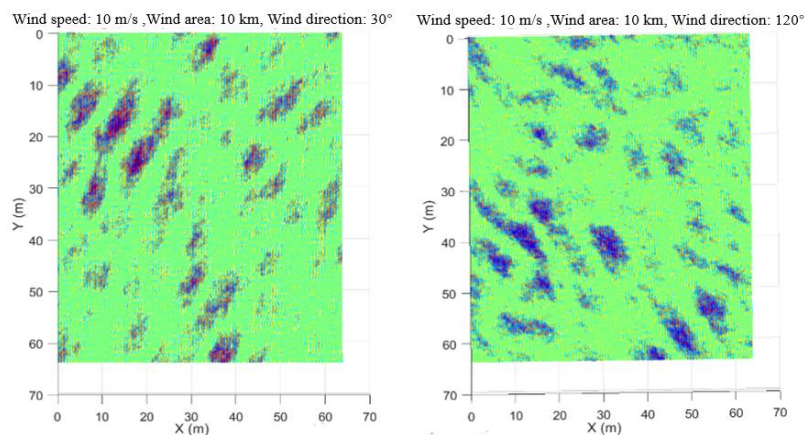


Fig. 4 ELH Sea Surface, $u_{10}=10\text{m/s}$, $\phi_w = 0^\circ$

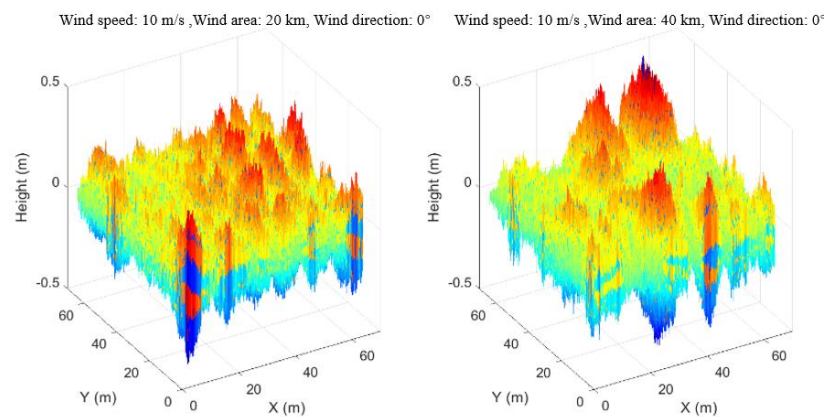
Using the two-dimensional Elfouhaily spectrum and linear filtering, the x- and y-axis intervals are set to 0.1 m, with a total length of 128 m, to model the sea surface. Fig. 5(a): With wind direction at 0° and fetch = 10 km, sea surfaces are generated under wind speeds of 5 m/s and 10 m/s. The vertical axis HHH denotes wave height. As wind speed increases, sea surface roughness becomes stronger. Fig. 5(b): With wind speed = 10 m/s, fetch = 10 km, and wind directions of 30° and 120° , sea surface top views are shown. Changes in wind direction do not significantly affect surface roughness. Fig. 5(c): With wind speed = 10 m/s, wind direction = 0° , and fetches of 20 km and 40 km, the results show that increasing fetch slightly enhances sea surface roughness.



(a) 2D Elfouhaily sea surface with different wind speeds



(b) 2D Elfouhaily sea surface top view with different wind directions



(c) 2D Elfouhaily sea surface with different wind areas

Fig. 5 Two-dimensional Elfouhaily sea surface models under different wind speeds, directions, and fetches

3.2 Superposition of Ship Wake and Sea Surface Geometric Models

Previous studies on ship wakes often neglected the ocean background. In reality, wakes are modulated by sea surface fluctuations. Existing models typically superimpose the three-dimensional wave heights of wake and sea surface directly at the same location^{[7]-[8]}. This method is simplistic and does not consider interactions between point sources of fluctuations. Therefore, a more realistic superposition approach is required.

This study proposes an approximate method: each point in the wake geometric model is treated as an energy point source, with its energy proportional to its fluctuation height. Greater height

corresponds to higher energy. The energy from each wake point acts on the surrounding sea surface, modifying the heights of nearby points in the sea surface model. Assuming the influence follows a Gaussian distribution, the two-dimensional Gaussian function is:

$$f(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \cdot \exp\left[-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_1)^2}{\sigma_1^2} - \frac{2\rho(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2} + \frac{(y-\mu_2)^2}{\sigma_2^2}\right)\right] \quad (26)$$

令 $\rho=0$, $\sigma_1=\sigma_2=\sigma$, $\mu_1=\mu_2=0$, 得到

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \quad (27)$$

令 $r^2=x^2+y^2$, 得

$$G(r) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \quad (28)$$

where σ is the standard deviation, adjustable based on the situation. As $r \rightarrow 0$, $G(r)$ reaches its maximum $1/2\pi\sigma^2$. In practice, when $r > r_0$, $G(r_0)=0.9$ is negligible, and its effect can be ignored. The contribution of a wake point to sea surface height is:

$$\Delta z = z_{\text{wake}} G(r) \quad (29)$$

where $G(r)$ is the weighting factor, z_{wake} is the wake height (can be negative), and z_{sea} is the sea surface height. The total height after superposition is:

$$z_{\text{total}} = z_{\text{sea}} + \Delta z \quad (30)$$

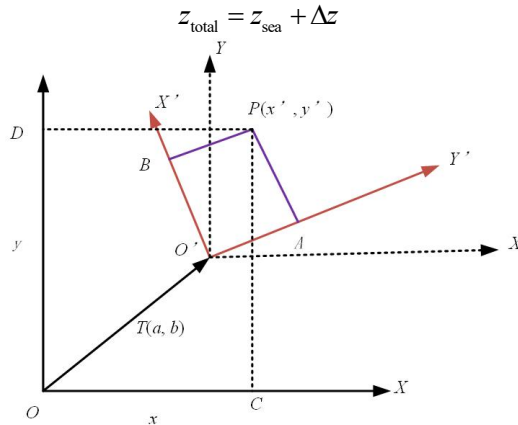
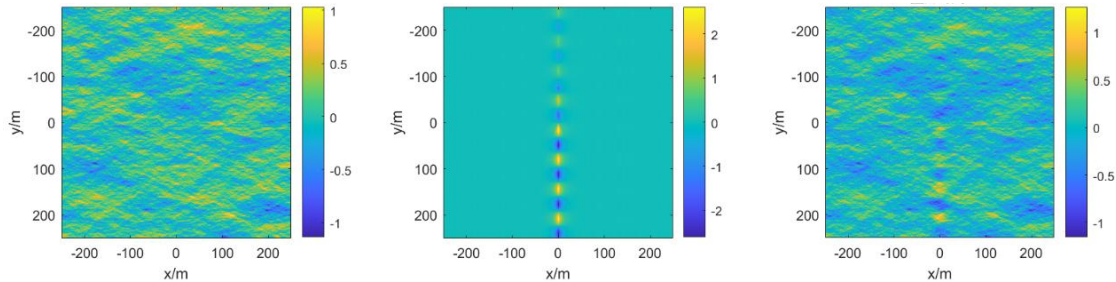


Fig. 6 Schematic diagram of coordinate transformation

Another advantage of this method is that by applying a coordinate transformation, the wake can be superimposed at any sea surface location. As shown in Fig. 7, the wake coordinates (x', y', z') in the local frame can be transformed to the sea surface coordinates (x, y, z) by:

$$\begin{cases} x = x' \cos \theta + y' \sin \theta + a \\ y = y' \cos \theta - x' \sin \theta + b \end{cases} \quad (31)$$



(a) Sea surface (b) Kelvin wake (c) Superposed result

Fig. 7 Superposition of Kelvin Wake and Sea Surface Geometric Models

As shown in Fig. 7, the sea surface significantly affects wake wave height, exerting a clear modulation effect.

4. Conclusions

This study investigated the coupling of ship wakes and sea surface geometric models and their impact on optical detection. The three-dimensional distribution laws of wake length, width, and depth were reviewed, confirming that Kelvin wakes exhibit sinusoidal periodic structures with fluctuation amplitude positively correlated with ship speed and hull size. Based on the Elfouhaily spectrum and Longuet-Higgins directional distribution function, an ELH sea surface model was constructed, revealing that wind speed is the key factor affecting sea surface roughness, while wind direction has no significant influence. To address the shortcomings of traditional direct superposition, a Gaussian energy-based method was proposed, achieving physical coupling through point-source energy and Gaussian weighting. Simulations confirmed that this approach accurately reproduces the modulation effect of the sea surface on wakes, providing theoretical support for improving the accuracy of optical detection of ship wakes.

5. Acknowledgment

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