

Development of a 6-DOF Fedundantly-Actuated 12-Legged Parallel Manipulator for Heavy-Load Applications

Yongzhi Chu^{1, 2, a}, Hasiaoqier Han^{1, 2, b *}, Tianjiao Ma^{1, 2, c}, Huze Huang^{1, 2, d},
Xiangquan Tan^{1, e}, Zhenbang Xu^{1, 2, f} and Qingwen Wu^{1, 2, g}

¹ Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences,
No.3888, Dong Nanhu Road, Changchun 130033, China;

² Center of Materials Science and Optoelectronics Engineering, University of Chinese Academy of
Sciences, Beijing 100049, China.

^a chuyongzhi@ciomp.ac.cn, ^b hanhasiaoqier@yahoo.com, ^c matianjiao@ciomp.ac.cn,

^d huuze@foxmail.com, ^e tanxq@ciomp.ac.cn, ^f xuzhenbang@ciomp.ac.cn, ^g wuqw@ciomp.ac.cn

Abstract. This paper presents the design and modeling of a new 6-degrees-of-freedom (DOF) 12-legged parallel manipulator (PM). A vector method is adopted to establish the kinematics model of the PM. The dynamics model is then developed using the principle of virtual work and the pseudo-inverse. Under equivalent conditions, the maximum driving force of the 6-DOF 12-legged PM is found to be only 55% of that of the traditional Gough–Stewart platform, indicating that the proposed PM achieves excellent dynamic performance and has huge load-bearing potential. In terms of control, a force/position hybrid control strategy is adopted to control the PM. The core of the position and force control systems are a proportional–integral–derivative (PID) controller and an interval type-2 fuzzy proportional–integral–derivative (IT2F-PID) controller, respectively. The correctness of the dynamic model and the effectiveness of the control method are verified through simulation analysis. The investigations of this paper provide new approaches for designing large-size and heavy-load PMs.

Keywords: Force/position hybrid control; heavy-load, IT2F-PID controller; parallel manipulator; redundantly-actuated.

1. Introduction

PMs are closed-loop motion chain mechanisms, in which the end effectors are connected to the base by several independent motion chains [1]. Compared with series manipulators, PMs offer high rigidity, high loading capacities, high accuracy, and good dynamic performance [2]. As a result, they are widely used in parallel machine tools [3], optical telescope precision pointing mechanisms [4], micro positioning systems [5], vibration isolation systems [6], seismic simulators [7], and motion simulators [8]. In some applications, the upper platform of a PM may require large dimensions or need to carry large loads. In such cases, traditional PMs typically have insufficient leg force and suffer deformation of the upper platform. To overcome these weaknesses, redundantly actuated parallel manipulators (RAPMs) have been developed. Due to the redundancy, RAPMs effectively avoid singular configurations in the workspace [9-10], increase the stiffness [11], expand the workspace, and greatly improve the loading capacities [12]. Under limited driving torque, adding redundant branches is an effective method of increasing the payload of manipulators and preventing deformation of the upper platform. It makes them more suitable for heavy-load applications [13].

However, everything has two sides. The RAPMs have some disadvantages. Actuation redundancy may result in an over-constrained mechanism. The Jacobian matrix of RAPMs, which maps the m -dimensional velocity vector $\dot{X}$ in the operating space into the n -dimensional joint velocity vector $\dot{q}$ is not a square matrix. This introduces difficulties to the kinematics and dynamics. In addition, RAPMs are multiple-input–multiple-output, nonlinear, and strongly coupled systems. It may generate constraints that lead to challenging control problems. Hence, the use of

position control alone may no longer be feasible [14]. Therefore, achieving actuation redundancy to improve the performance of PMs, is a very interesting issue, especially for heavy-load and large-size applications. This is one of the primary motivations for this study. In the case of heavy-load applications, RAPMs often use additional actuation legs on non-redundant structures to increase their load capacities [11-13,15]. To address the control challenges of RAPMs, model-based PD controllers [16], position and force switching control strategies [11], decoupling sliding mode control [17], and force/position hybrid control strategies [18-20] have been developed.

This article describes a 6-DOF 12-legged RAPM. On the basis of the Gough-Stewart platform [21], six additional actuation legs are added to form a 6-DOF 12-legged RAPM. This formulation greatly improves the carrying capacity of the PM without increasing the output forces of legs. Through numerical analysis, it is found that the maximum leg driving force of the 12-legged RAPM is only 55% of that of a traditional PM under the same working conditions. In addition, due to the increase of legs, the force on the upper platform is more uniform, creates less stress, and generates smaller deformation. In terms of control, a hybrid force/position control method is used. Based on a PID controller, a position control system is designed for the 1st to 6th legs. The force control branch of the 7th to 12th legs is an optimal IT2F-PID controller. These control schemes ensure that the requirements of the 6-DOF 12-legged RAPM are satisfied.

The remainder of this paper is organized as follows. In Section 2, the proposed RAPM is described in detail. Analysis of the kinematics and dynamics is conducted in Section 3. The control design is developed in Section 4, and simulation results are presented in Section 5. Finally, the conclusions to this study are summarized in Section 6.

2. Structure

The architecture drawing of the 6-DOF 12-legged RAPM is shown in Fig. 1(a). The RAPM is composed of a mobile platform, a fixed platform and twelve legs with identical structures. The mobile platform is connected to the base platform through the legs. The ends of all legs are universal joints (U). As shown in Fig. 1(b), the upper and lower connection points are $P_i(i=1,2,\dots,12)$ and $B_i(i=1,2,\dots,12)$, respectively.

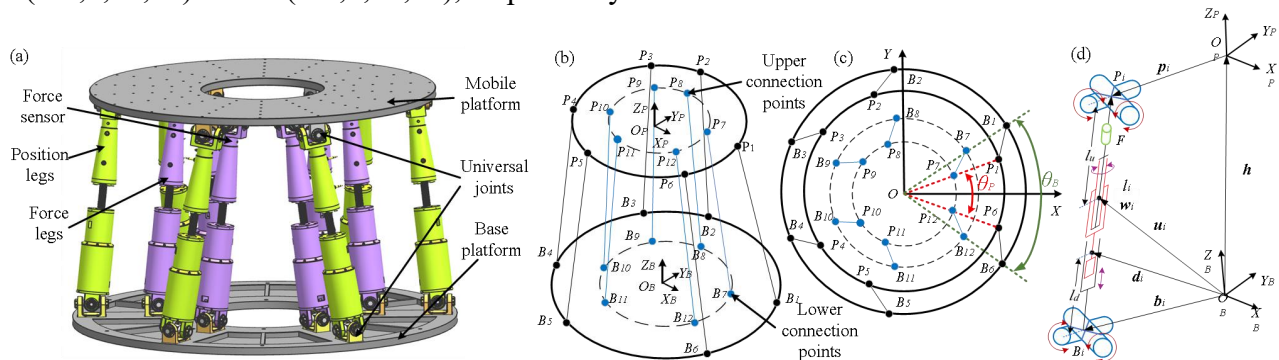


Fig. 1. Schematic of the 6-DOF 12-legged RAPM: (a) the architecture drawing of the PM, (b) geometric model of the PM, (c) vertical view of the PM, (d) vector diagram of a leg of URPU kinematic chain.

Among the legs, six are arranged in the inner circle, and the other six are arranged in the outer circle. The centers of U on the inner or outer legs are configured as a symmetric hexagon about the x-axis. The angle is θ_P between the centers. The configuration of the lower U centers is consistent with that of the upper U, and the angle between the centers is θ_B , as shown in Fig. 1(c). Each leg is driven by an alternating current motor through a revolute and prismatic kinematic chain (RP), as shown in Fig. 1(d). The force sensor (F) is installed between U and RP to achieve force/position hybrid control. Based on the Kutzbach–Grubler formula [22], the number of DOFs of the mechanism is calculated as

$$M = 6(n - g - 1) + \sum_{i=1}^g f_i, \quad (1)$$

where M is the number of DOFs, n is the number of members, g is the number of kinematic pairs, and f_i is the number of DOFs of the motion pair. Analysis of the RAPM shows that $n=50$, $g=60$, and $\sum_{i=1}^g f_i=72$. Thus, the number of DOFs is six.

3. Kinematics and Dynamics

3.1 Kinematics

The following coordinate system is used to analyze the kinematics. The global reference frame $O_B-X_B Y_B Z_B$ is located on the bottom surface of the base platform. The body reference frame $O_P-X_P Y_P Z_P$ is located on the upper surface of the mobile platform. In practical applications, the position, velocity, and acceleration vectors of the mobile platform relative to its own body reference frame $O_P-X_P Y_P Z_P$ are provided. Let $\mathbf{h} = [X \ Y \ Z]^T$ be the position vector and $\mathbf{t} = [\alpha \ \beta \ \gamma]^T$ be the attitude vector. The pose of a mobile platform is represented as a vector $\mathbf{q} = [\mathbf{h}^T \ \mathbf{t}^T]^T$. The generalized speed $\dot{\mathbf{q}}$ of a mobile platform is represented in the $O_B-X_B Y_B Z_B$: $\dot{\mathbf{q}} = [\dot{\mathbf{h}}^T \ \dot{\boldsymbol{\omega}}^T]^T$ where the angular velocity vector $\boldsymbol{\omega}$ is obviously not equal to $\dot{\mathbf{t}}$. The $\boldsymbol{\omega}$ of the mobile platform can be obtained from the vector $\dot{\mathbf{t}}$ as [2] $\boldsymbol{\omega} = {}^B_R \cdot E \cdot \dot{\mathbf{t}}$.

Position analysis is the basis of kinematics. As shown in Fig. 1(d), the closed-loop position equation related to the kinematic chain of the i th leg ($i=1, 2, \dots, 12$) can be written as

$$l_i \mathbf{w}_i = \mathbf{h} + \mathbf{p}_i - \mathbf{b}_i, \quad (2)$$

where l_i represents the length, $\mathbf{w}_i$ represents the unit vector along the leg axis, $\mathbf{b}_i$ represents the length vector of $O_B B_i$, $\mathbf{h}$ represents the length vector of $O_B O_P$, and $\mathbf{p}_i$ represents the length vector of $O_P P_i$. The length of the legs obtained by dot multiplying both sides of Eq. (2) with $\mathbf{w}_i$:

$$l_i = (\mathbf{h} + \mathbf{p}_i - \mathbf{b}_i) \cdot \mathbf{w}_i, \quad (3)$$

As shown in Fig. 1(d), l_u and l_d are the distances from P_i and B_i to the centroid of the upper and lower leg, respectively. The following equations describe the centroid positions of the upper and lower leg cylinders: $\mathbf{u}_i = \mathbf{b}_i + (l_i - l_u) \mathbf{w}_i$, $\mathbf{d}_i = \mathbf{b}_i + l_d \mathbf{w}_i$. The velocity V_{P_i} of P_i can be obtained by taking the time derivative of the right-hand and left-hand sides of Eq. (2):

$$\mathbf{V}_{P_i} = \dot{\mathbf{h}} + \boldsymbol{\omega} \times \mathbf{p}_i, \quad (4)$$

$$\mathbf{V}_{P_i} = \dot{l}_i \mathbf{w}_i + l_i \boldsymbol{\omega}_i \times \mathbf{w}_i, \quad (5)$$

where $\dot{\mathbf{h}}$ is the velocity vector of point O_P . The elongation velocity and angular velocity vector of the i th leg are $\dot{l}_i$ and $\boldsymbol{\omega}_i$, respectively. Using $\mathbf{w}_i$ to dot multiply Eqs. (4) and (5), omitting the angular velocity of the i th leg rotating around its axis, $\dot{l}_i$ is obtained as

$$\dot{l}_i = \mathbf{w}_i \cdot \dot{\mathbf{h}} + \mathbf{w}_i \cdot (\boldsymbol{\omega} \times \mathbf{p}_i), \quad (6)$$

Cross multiplying Eqs. (8) and (9) with $\mathbf{w}_i$ yields. Considering $\mathbf{w}_i \times \mathbf{w}_i = 0$, $\mathbf{w}_i \cdot \mathbf{w}_i = 1$, and $\boldsymbol{\omega}_i \cdot \mathbf{w}_i = 0$, the angular velocity vector of the i th leg is obtained as

$$\boldsymbol{\omega}_i = \frac{1}{l_i} (\mathbf{w}_i \times \dot{\mathbf{h}}) + \frac{1}{l_i} \mathbf{w}_i \times (\boldsymbol{\omega} \times \mathbf{p}_i), \quad (7)$$

Derive the centroid velocities of the upper and lower leg cylinders by taking the time derivative of $\mathbf{u}_i$ and $\mathbf{d}_i$:

$$\mathbf{V}_{u_i} = \dot{l}_i \mathbf{w}_i + (l_i - l_u) \boldsymbol{\omega}_i \times \mathbf{w}_i, \quad (8)$$

$$\mathbf{V}_{d_i} = \boldsymbol{\omega}_i \times l_d \mathbf{w}_i, \quad (9)$$

Same as pushing displacement to velocity, the elongation acceleration of the leg $\ddot{l}_i$, the angular acceleration vector of the leg $\ddot{\omega}_i$, and the centroid acceleration of the upper and lower leg V_{ui} and V_{di} can be derived. The elongation velocity matrix of the i th leg in the $O_B-X_B Y_B Z_B$ frame is obtained from Eq. (6) as

$$\dot{l}_i = [\mathbf{w}_i \quad \mathbf{p}_i \times \mathbf{w}_i] \begin{bmatrix} \dot{\mathbf{h}} \\ \dot{\omega} \end{bmatrix} = [{}^B \mathbf{w}_i^T \quad ({}^B \tilde{\mathbf{p}}_i {}^B \mathbf{w}_i)^T] \begin{bmatrix} \dot{\mathbf{h}} \\ \dot{\omega} \end{bmatrix} = {}^B J_i \dot{\mathbf{q}}, \quad (10)$$

where ${}^B \mathbf{w}_i$ is the matrix of $\mathbf{w}_i$, ${}^B \tilde{\mathbf{p}}_i$ is the antisymmetric matrix of ${}^B \mathbf{p}_i$, and ${}^B \mathbf{p}_i$ is the length vector matrix of P_i . According to Eq. (7), the angular velocity array of the i th leg in the $O_B-X_B Y_B Z_B$ is

$${}^B \omega_i = \frac{1}{l_i} [{}^B \mathbf{w}_i \quad -{}^B \tilde{\mathbf{w}}_i {}^B \tilde{\mathbf{p}}_i] \begin{bmatrix} \dot{\mathbf{h}} \\ \dot{\omega} \end{bmatrix} = {}^B J_i \dot{\mathbf{q}}, \quad (11)$$

where ${}^B \tilde{\mathbf{w}}_i$ is the antisymmetric matrix of ${}^B \mathbf{w}_i$. According to Eqs. (8) and (9), the centroid velocities of the upper and lower legs are

$${}^B V_{ui} = \begin{bmatrix} {}^B \mathbf{w}_i {}^B \mathbf{w}_i^T - \frac{l_i - l_u}{l_i} {}^B \tilde{\mathbf{w}}_i {}^B \tilde{\mathbf{w}}_i \\ -{}^B \mathbf{w}_i ({}^B \tilde{\mathbf{w}}_i {}^B \mathbf{p}_i)^T + \frac{l_i - l_u}{l_i} {}^B \tilde{\mathbf{w}}_i {}^B \tilde{\mathbf{w}}_i {}^B \tilde{\mathbf{p}}_i \end{bmatrix}^T \begin{bmatrix} \dot{\mathbf{h}} \\ \dot{\omega} \end{bmatrix} = {}^B J_{ui} \dot{\mathbf{q}}, \quad (12)$$

$${}^B V_{di} = -\frac{l_d}{l_i} [{}^B \tilde{\mathbf{w}}_i {}^B \tilde{\mathbf{w}}_i \quad -{}^B \tilde{\mathbf{w}}_i {}^B \tilde{\mathbf{w}}_i {}^B \tilde{\mathbf{p}}_i] \begin{bmatrix} \dot{\mathbf{h}} \\ \dot{\omega} \end{bmatrix} = {}^B J_{di} \dot{\mathbf{q}}, \quad (13)$$

3.2 Dynamics

Regarding the inverse dynamics, the input force required for the legs to generate the motion is determined based on the expected trajectory, the external forces and moments on the mobile platform. The dynamic equation is constructed using the principle of virtual work [23]. When the external forces, moments, inertia forces, and inertia moments acting on the RAPM are applied, the RAPM is in a balanced state. Regardless of the frictional torque at P_i and hinge B_i , the frictional force between the upper and lower legs and the rotation around the axis of the upper legs are omitted. The dynamic equation expressed by the principle of virtual work is

$$\sum_{i=1}^{12} f_{Di} \delta l_i + \begin{bmatrix} {}^B \mathbf{F}_P + {}^B \mathbf{G}_P \\ {}^B \mathbf{M}_P \end{bmatrix}^T \delta {}^B \mathbf{q} + \begin{bmatrix} -m_p {}^B \ddot{\mathbf{h}} \\ -{}^B I_p \cdot {}^B \dot{\omega} - {}^B \tilde{\omega} {}^B I_p {}^B \omega \end{bmatrix}^T \delta {}^B \mathbf{q} + \sum_{i=1}^{12} ({}^B \mathbf{G}_{ui} - m_{ui} {}^B \dot{V}_{ui}) \delta {}^B l_{ui} + \sum_{i=1}^{12} (-{}^B I_{ui} \cdot {}^B \dot{\omega}_i - {}^B \tilde{\omega}_i {}^B I_{ui} {}^B \omega_i) \delta {}^B \theta_i + \sum_{i=1}^{12} ({}^B \mathbf{G}_{di} - m_{di} {}^B \dot{V}_{di}) \delta {}^B l_{di} + \sum_{i=1}^{12} (-{}^B I_{di} \cdot {}^B \dot{\omega}_i - {}^B \tilde{\omega}_i {}^B I_{di} {}^B \omega_i) \delta {}^B \theta_i = 0, \quad (14)$$

where $f_{Di} \delta l_i$ is the virtual work of the driving force f_{Di} applied to the i th leg, $\begin{bmatrix} {}^B \mathbf{F}_P + {}^B \mathbf{G}_P \\ {}^B \mathbf{M}_P \end{bmatrix}^T \delta {}^B \mathbf{q}$ is the virtual work of the forces and moments on the mobile platform, $\begin{bmatrix} -m_p {}^B \ddot{\mathbf{h}} \\ -{}^B I_p \cdot {}^B \dot{\omega} - {}^B \tilde{\omega} {}^B I_p {}^B \omega \end{bmatrix}^T \delta {}^B \mathbf{q}$ is the virtual work of the inertia forces and moments on the mobile platform, $({}^B \mathbf{G}_{ui} - m_{ui} {}^B \dot{V}_{ui}) \delta {}^B l_{ui}$ and $({}^B \mathbf{G}_{di} - m_{di} {}^B \dot{V}_{di}) \delta {}^B l_{di}$ are the virtual work of the external and inertial forces on the centroids of the upper and lower leg, $(-{}^B I_{ui} \cdot {}^B \dot{\omega}_i - {}^B \tilde{\omega}_i {}^B I_{ui} {}^B \omega_i) \delta {}^B \theta_i$ and $(-{}^B I_{di} \cdot {}^B \dot{\omega}_i - {}^B \tilde{\omega}_i {}^B I_{di} {}^B \omega_i) \delta {}^B \theta_i$ are the virtual work of the inertia moment on the upper and lower leg. Assuming $\mathbf{f}_D = [f_{D1} \quad f_{D2} \quad \cdots \quad f_{D12}]^T$, $\sum_{i=1}^{12} f_{Di} \delta l_i = \mathbf{f}_D^T \delta \mathbf{l}$. Converting the inverse velocity Jacobian matrix into a force Jacobian matrix the Eq. (19) yields

$${}^B J^T f_D + \begin{bmatrix} {}^B F_P + {}^B G_P \\ {}^B M_P \end{bmatrix} + \begin{bmatrix} -m_p {}^B \ddot{h} \\ -{}^B I_p \cdot {}^B \dot{\omega} - {}^B \tilde{\omega} {}^B I_p {}^B \omega \end{bmatrix} + \sum_{i=1}^{12} {}^B J^T ({}^B G_{ui} - m_{ui} {}^B \dot{V}_{ui}) + \sum_{i=1}^{12} {}^B J^T (-{}^B I_{ui} \cdot {}^B \dot{\omega}_i - {}^B \tilde{\omega}_i {}^B I_{ui} {}^B \omega_i) + \sum_{i=1}^{12} {}^B J^T ({}^B G_{di} - m_{di} {}^B \dot{V}_{di}) + \sum_{i=1}^{12} {}^B J^T (-{}^B I_{di} \cdot {}^B \dot{\omega}_i - {}^B \tilde{\omega}_i {}^B I_{di} {}^B \omega_i) \delta^B \theta_i = 0, \quad (15)$$

The pseudo inverse ${}^B J^{*-T}$ of ${}^B J^T$ is multiplied by both sides of Eq. (15) to yield the driving force of the 12 legs:

$$f_D = -{}^B J^{*-T} \begin{bmatrix} {}^B F_P + {}^B G_P \\ {}^B M_P \end{bmatrix} + {}^B J^{*-T} \begin{bmatrix} m_p {}^B \ddot{h} \\ {}^B I_p \cdot {}^B \dot{\omega} + {}^B \tilde{\omega} {}^B I_p {}^B \omega \end{bmatrix} - {}^B J^{*-T} \sum_{i=1}^{12} {}^B J^T ({}^B G_{ui} - m_{ui} {}^B \dot{V}_{ui}) + {}^B J^{*-T} \sum_{i=1}^{12} {}^B J^T ({}^B I_{ui} \cdot {}^B \dot{\omega}_i + {}^B \tilde{\omega}_i {}^B I_{ui} {}^B \omega_i) - {}^B J^{*-T} \sum_{i=1}^{12} {}^B J^T ({}^B G_{di} - m_{di} {}^B \dot{V}_{di}) + {}^B J^{*-T} \sum_{i=1}^{12} {}^B J^T ({}^B I_{di} \cdot {}^B \dot{\omega}_i + {}^B \tilde{\omega}_i {}^B I_{di} {}^B \omega_i), \quad (16)$$

4. Controller design

From Eq. (16), it is clear that RAPMs are complex time-varying, highly nonlinear systems with strong coupling characteristics. Under heavy-load and high-speed conditions, the equipment may be damaged by branch errors and opposite reaction forces. Force/position hybrid control is a common scheme for RAPMs [20]. The control structure diagram is shown in Fig. 2.

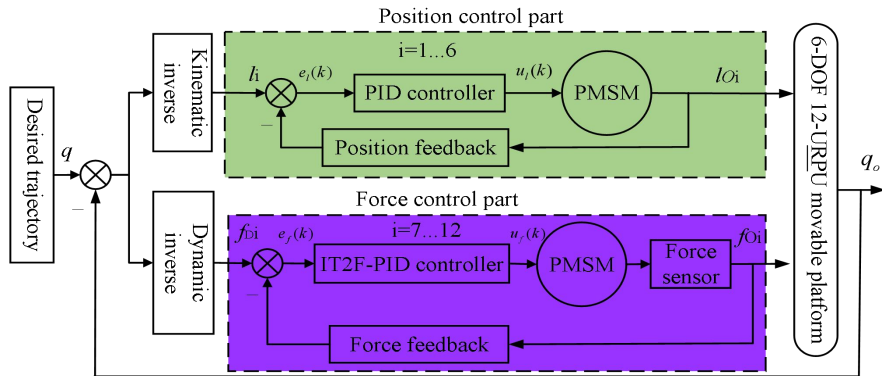


Fig. 2. The control diagram of 6-DOF 12-legged RAPM.

4.1 Position control design

The legs are driven by an AC system. The simple structure and easy implementation of three-loop control makes it common in AC servo systems. So, legs 1–6 adopt a three-loop position control scheme. The position control block diagram is shown in Fig. 3,

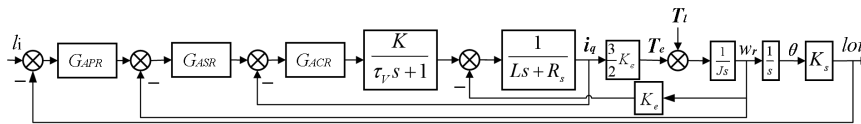


Fig. 3. The diagram of position control.

where G_{APR} , G_{ASr} , and G_{ACR} represent the position loop, speed loop, and current loop controllers, respectively ¹⁴. $\frac{K}{\tau_v s + 1}$ is the simplified transfer function of a pulse-width modulation inverter, where K and τ_v represent the amplification factor of the inverter and the time constant, respectively. $\frac{1}{Ls + Rs}$ is the simplified transfer function of a permanent magnet synchronous motor (PMSM), where L and R are the armature inductance and the stator resistance, respectively. K_e is the motor torque

coefficient, K_s is the ratio coefficient of the ball screw and the reducer, and J is the rotational inertia of the motor rotor and screw. G_{ASR} and G_{ACR} are PI controllers, whereas G_{APR} is a PID controller. The l_i is obtained from Eq. (10). The structure of the PID controller is as follows [25]:

$$u_l(k) = k_p e_l(k) + k_i \int e_l(k) dk + k_d \frac{de_l(k)}{dk}$$

$$e_l(k) = l_i - l_{oi} \quad (17)$$

where k_p , k_i , and k_d are the proportional, integral, and derivative gains, respectively. $u_l(k)$, l_i , l_{oi} , and $e_l(k)$ are the controller output, expected target value, system output value, and difference between the target value and system output value, respectively.

4.2 Force control design

Legs 7–12 are controlled by a force control scheme. The force control diagram is shown in Fig. 4,

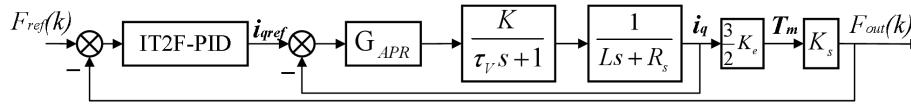


Fig. 4. The diagram of force control.

where G_{FACR} is the current loop controller, which is a PI controller. The other parameters are the same as for the position control scheme. The expected value of f_{Di} is obtained from Eq. (26). The force controller is an IT2F-PID controller, as shown in Fig. 5.

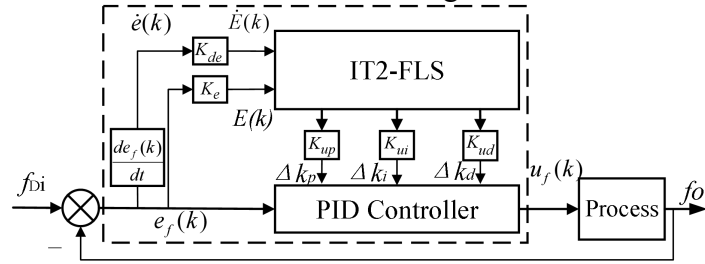


Fig. 5. IT2F-PID controller structure.

The controller consists of an upper-level interval type-2 fuzzy logic system (IT2-FLS) and a lower-level PID controller[25]. The IT2-FLS provides a parameter change mechanism for the lower controller, and the PID controller parameters are adjusted to control the system. For the IT2F-PID controller, a PID controller is first designed, and then the IT2-FLS is developed. According to experience, k_p , k_i , and k_d are set to 0.04, 222, and 0, respectively. The IT2-FLS consists of four parts: a fuzzifier, an inference engine, a rule base, and an output processor. The working principle of the IT2-FLS is as follows[27]. The clear inputs $e_f(k)$ and $de_f(k)/dt$ from the input sensor are projected into the fuzzy sets and fuzzified into interval type-2 fuzzy sets (IT2-FSs). Based on experience, E , $\dot{E}$, U_p , U_i , and U_d are projected into three levels, namely: "positive (P)", "zero (ZO)", and "negative (N)". The relationships between the fuzzy language values E , $\dot{E}$, U_p , U_i , U_d and actual input-output values e , $\dot{e}$, Δk_p , Δk_i , Δk_d is shown in Eq. (22)

$$\begin{cases} E = K_e \cdot e \\ \dot{E} = K_{de} \cdot \dot{e} \\ \Delta k_p = K_{up} \cdot U_p \\ \Delta k_i = K_{ui} \cdot U_i \\ \Delta k_d = K_{ud} \cdot U_d \end{cases} \quad (18)$$

The membership function is illustrated in Fig. 6(a). This is a trigonometric function with a range of $[-1, 1]$. The parameters of N are $[-1, -1, -0.2]$, the parameters of ZO are $[-0.8, 0, 0.8]$, and the parameters of P are $[0.2, 1, 1]$.

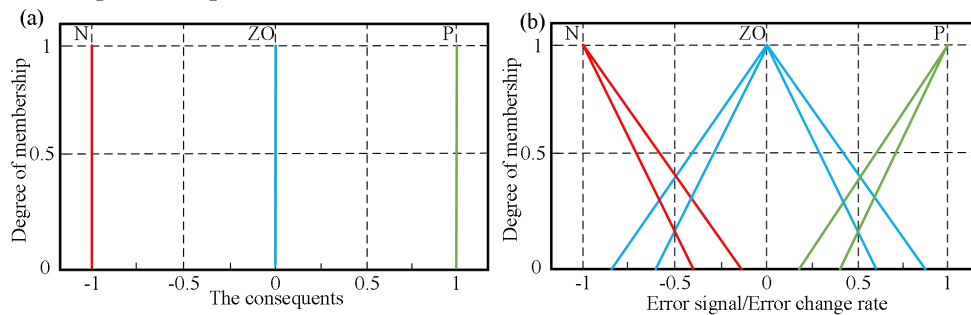


Fig. 6. Membership functions of the parameters: (a) IT2-FLS for E and $\dot{E}$, (b) U_p , U_i , and U_d .

A rule library is designed according to the impact of k_p , k_i , and k_d on the control system. Based on Table 1 and experience, the fuzzy rules are as listed in Tables 2–4.

Table 1. Effects of gain parameters

Gain parameters	Rise time	Overshoot	Settling time
k_p	Decrease	Increase	Small change
k_i	Decrease	Increase	Increase
k_d	Decrease	Decrease	Decrease

Table 2. Rule base for the proportional gain

Error signal	N	ZO	P
N	N	N	N
ZO	N	P	P
P	P	P	P

Table 3. Rule base for the integral gain

Error signal	N	ZO	P
N	ZO	ZO	ZO
ZO	P	P	P
P	ZO	ZO	ZO

Table 4. Rule base for the derivative gain

Error signal	N	ZO	P
N	N	N	P
ZO	N	ZO	P
P	N	N	P

The membership functions of the consequents for U_p , U_i , and U_d are single values, as shown in Fig. 6(b). The corresponding values of $\{N, ZO, P\}$ are $\{-1, 0, 1\}$. The fuzzy output of the inference engine is processed by a type reducer. In this study, the center-of-set method based on the Karnik–Mendel algorithm is selected to map the IT2-FSs to T1-FSs [28]. The clear output values (Δk_p , Δk_i , and Δk_d) are then obtained, which are the adjustment values relating to k_p , k_i , and k_d .

5. Numerical simulations

5.1 Dynamics simulation

To verify the effectiveness of the dynamics, the results of the dynamics model are compared with simulation software results. The parameters of the RAPM are listed in Table 5.

Table 5. The parameters of the parallel manipulator.

Parameters	Value	Parameters	Value
R_{PI}	0.225m	R_{PO}	0.4m
R_{BI}	0.275m	R_{BO}	0.45m
θ_P	40°	θ_B	60°
H	0.534m	${}^B F_P$	(100, 100, -10000) N
m_p	100	${}^B M_P$	(100, 100, 100) N·m
l_d	0.13812m	l_u	0.12334m
$m_{di} (i=1,2,...,12)$	9.206kg	$m_{ui} (i=1,2,...,12)$	3.393kg

R_{PI} , R_{PO} , R_{BI} , and R_{BO} are the distribution circle radii of points P_i ($i=7, 8, \dots, 12$), P_i ($i=1, 2, \dots, 6$), B_i ($i=7, 8, \dots, 12$), and B_i ($i=1, 2, \dots, 6$), respectively. H is the distance between the distribution planes of the upper and lower hinge points. ${}^B F_P$ and ${}^B M_P$ represent the force and torque on the upper platform, respectively. The expected posture on the platform is $x = 0.002 \sin(t)$, $y = 0.003 \cos(t)$, $z = 0.004 \sin(t)$, $\alpha = \frac{0.02}{180} \pi \sin(t)$, $\beta = \frac{0.03}{180} \pi \sin(t)$, $\gamma = \frac{0.04}{180} \pi \sin(t)$ where $t \in [0, 2\pi]$. Here, x , y , and z are the distances traveled by the mobile platform along the X , Y , and Z axes, respectively. α , β , and γ are the rotation angles of the mobile platform around the X , Y , and Z axes, respectively. The force and posture of the mobile platform are given, and the output force of each leg is calculated using the dynamics model and the simulation software, respectively. The two results are compared to verify the correctness of the dynamics. Due to the force/position hybrid control, legs 7–12 are controlled by force. Therefore, the driving force calculated by the dynamics is input to legs 7–12 in the simulation software. The driving force of legs 1–6 is compared, as shown in Fig. 7(a). The driving forces of the legs calculated by the dynamics model (F1–F6) are consistent with the results from the simulation software (F'1–F'6), indicating that the dynamics model is accurate.

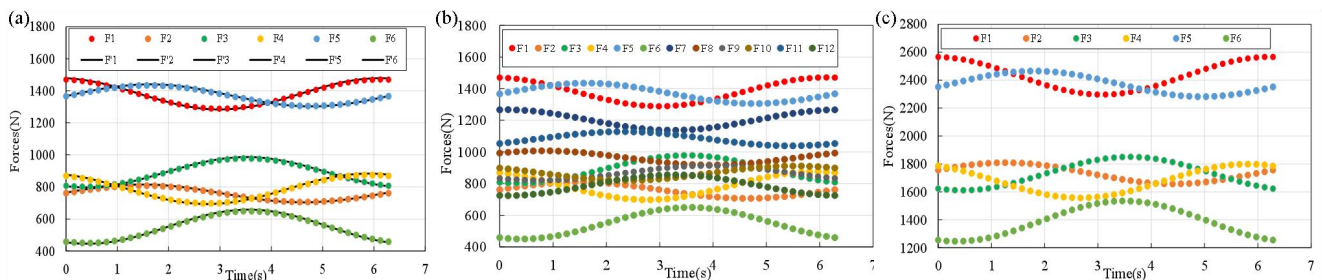


Fig. 7. Variations of the leg forces: (a) comparison of the dynamics model and software, (b) the 6-DOF 12-legged RAPM, (c) the non-redundant PM.

To illustrate the dynamic performance of the RAPM, the driving forces of the legs are used as the evaluation indicator. Under the same working conditions, the leg driving forces of redundant and general PMs are compared. The two types of manipulators have the same structure, but different numbers of legs. As shown in Fig. 7(b) and Fig. 7(c), the driving force of the RAPM is significantly less (by 45%) than that of the general PM. In addition, the maximum difference in the leg forces of the RAPM is only 75% of that of the general PM, which means that the support force on the upper platform is more uniform and the deformation is smaller. Further clarification, the proposed structure has significant advantages in large-size and heavy-load applications.

5.2 Control simulation

To verify the effectiveness of the proposed control system, the position and force controllers are connected to a virtual prototype of the RAPM and a series of simulations are conducted. The motor and control system data are presented in Table 6.

Table 6. The parameters of the parallel manipulator.

Parameters	Meaning	Value	Unit
L	Rotator inductance	0.01225	H
R	Rotator resistance	12.4	Ω
Pn	Number of pairs	5	/
Ke	Torque constant	0.1125	/
τ_V	Inverter time constant	0.001	s
K	Inverter gain	1	/
Ks	Proportionality coefficient	$1/400\pi$	/
J	Movement of initial	0.000188	$\text{Kg}\cdot\text{m}^2$
K_{pi}	The proportional parameter of G_{ACR}	109	/
K_{ii}	The integral parameter of G_{ACR}	1	/
K_{pv}	The proportional parameter of G_{ACR}	1.2	/
K_{iv}	The integral parameter of G_{ASR}	13.5	/
K_{pp}	The proportional parameter of G_{ASR}	25	/
K_{ip}	The integral parameter of G_{APR}	500	/
K_{dp}	The derivative parameter of G_{APR}	0.025	/
K_{pif}	The proportional parameter of G_{FACR}	39	/
K_{iif}	The integral parameter of G_{FACR}	8563	/
K_e	The scale factor of e	0.001	/
K_{de}	The scale factor of $\dot{e}$	3.5×10^{-7}	/
K_{up}	The scale factor of U_p	0.076	/
K_{ui}	The scale factor of U_i	54.4	/
K_{ud}	The scale factor of U_d	9.4×10^{-6}	/

The simulation results are shown in Fig. 8 and Fig. 9. The position response curve of the legs (Fig. 8) does not exceed the overshoot and the stabilization time is short. Additionally, there is no steady-state error after stabilization. This demonstrates that the designed position control fully satisfies the requirements.

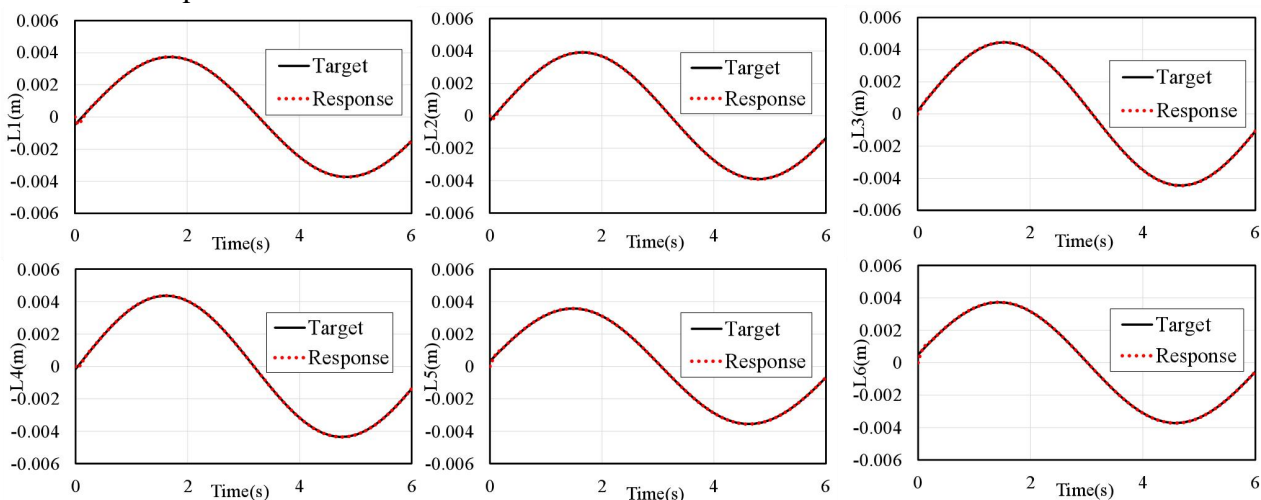


Fig. 8. The response of legs position control.

As shown in Fig. 9, the response of the leg force exhibits a slight overshoot, but quickly stabilizes after a brief oscillation without steady-state errors. This verifies that the designed force control method satisfies the requirements. Comparing Fig. 9 with Fig. 7(a) illustrates that the control simulation results are consistent with the dynamic calculation results, further indicating that the designed PM is fully achievable.

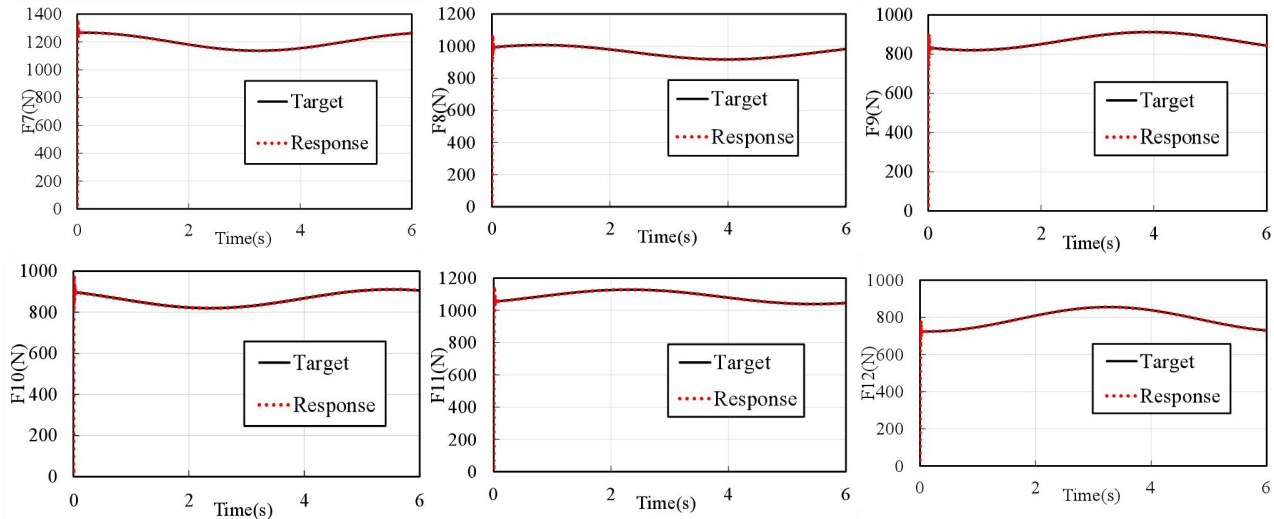


Fig. 9. The response of legs force control.

6. Conclusion

A new 6-DOF 12-legged RAPM has been introduced. The PM has 12 driving legs arranged in inner and outer circles, which increases the load-bearing capacity without increasing the driving force of the legs. The RAPM kinematics model was established using a vector method. The position, velocity, and acceleration and kinematics equations of the whole machine were solved. The dynamics were then established using the principle of virtual work. The driving force equations of the legs were solved using the pseudo-inverse. To verify the accuracy of the kinematics and dynamics models, the calculation results were compared with those from simulation software. The two results were consistent, indicating that the kinematics and dynamics are correct. To verify the dynamic performance of the RAPM, the driving force of the legs was investigated as an evaluation indicator. Under the same conditions, the maximum leg driving force of the RAPM was found to be only 55% of that of a traditional non-redundant PM, indicating that the designed RAPM achieves better dynamic performance.

In terms of control, a force/position hybrid control method was developed, with position control for legs 1–6 and force control for legs 7–12. The position control system is based on a PID controller and the force controller system is based on an IT2F-PID controller. To verify the effectiveness of the control system, the position and force controllers were connected to a virtual prototype of the RAPM and simulations were conducted. The simulation results show that the position and force tracking performance of each leg meet the usage requirements. The results presented in this paper provide theoretical support for subsequent prototype production. It also provides new ideas for large-size and heavy-load PMs.

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